COHERENT PARTICLE ENERGIZATION BY ELECTROSTATIC WAVES

D. J. Strozzi[1], A. K. Ram, A. Bers

Plasma Science and Fusion Center
Massachusetts Institute of Technology
Cambridge, MA 02139. U.S.A.

Supported by: DOE Contract DE-FG02-91ER-54109 NSF Contract ATM-0114462, DOE/NSF Contract DE-FG02-99ER-54555, Princeton Univ. Subcontract 150-6804-1

2003 International Sherwood Theory Conference
Corpus Christi, TX
Poster 2C27
29 April 2003

[1]email: dstrozzi@mit.edu. Poster available at www.mit.edu/~dstrozzi

Abstract

Heating particles in a background magnetic field by electrostatic waves is of interest in both laboratory and space plasmas. It can be achieved by a single perpendicular or oblique wave driving particles into stochastic dynamics [2]. However, low-energy particles can only be heated with a large-amplitude wave.

Unlike for a single wave, multiple waves allow for coherent energization of low-energy particles. For example, this occurs due to two perpendicular waves[3] whose frequencies ω_1, ω_2 differ by a multiple of the particle cyclotron frequency ω_c: $\omega_1 - \omega_2 = N\omega_c$. Such a process may explain the high-energy tail of H^+ and O^+ distributions in the upper ionosphere [4].

Since the coherent energization does not require a minimum field amplitude, waves with arbitrarily small amplitudes can heat low-energy particles. The timescale for coherent energization scales as the fourth power of the wave frequencies and inversely as the wave amplitudes squared. Waves with finite parallel wavenumbers $k_\parallel$ produce coherent perpendicular energization provided $k_{1\parallel}, k_{2\parallel}$ are sufficiently close (regardless of their magnitudes).

Coherent energization also occurs in a spectrum of waves if enough waves satisfy the frequency-matching condition. Electric fields with a continuous frequency spectrum vary in amplitudes over time. A slow change in amplitude affects the rate of energization.

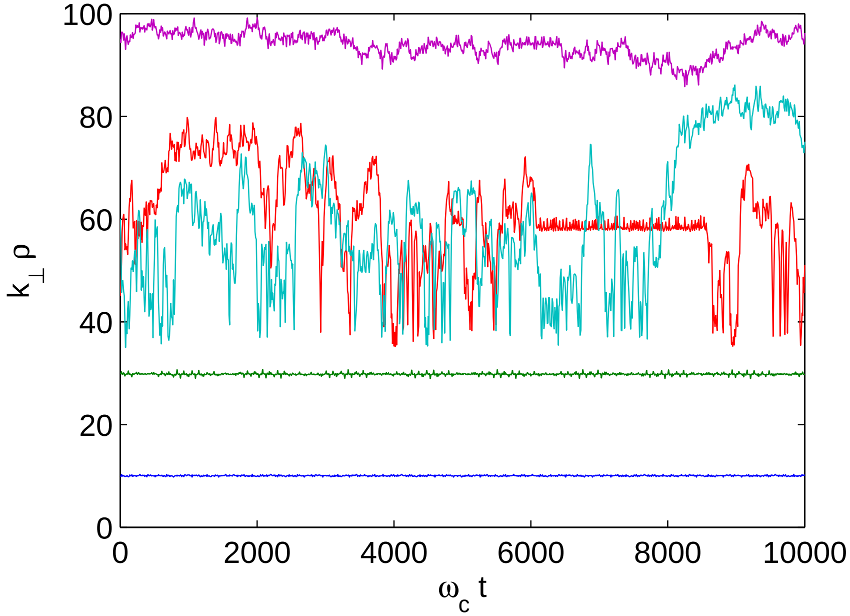
Waves below or near ω_c can also coherently energize particles if $\omega_1 + \omega_2 = N\omega_c$. This heating occurs regardless of the wave amplitudes, in contrast to the amplitude-dependent stochastic heating with waves below ω_c reported by Chen *et al.* [5].

[2] C. Karney, *Phys. Fluids* 21,9 (1978), G. Smith, A. Kaufman, *Phys. Fluids* 21,12 (1978)
[3] D. Bénisti, A. K. Ram, A. Bers, *Phys. Plasmas*, 5,9 (1998)
[4] A. K. Ram, A. Bers, D. Bénisti, *J. Geophys. Res.*, 103,A5 (1998)
[5] L. Chen, Z. Lin, R. White, *Phys. Plasmas* 8,11 (2001)

Motivation and Outline

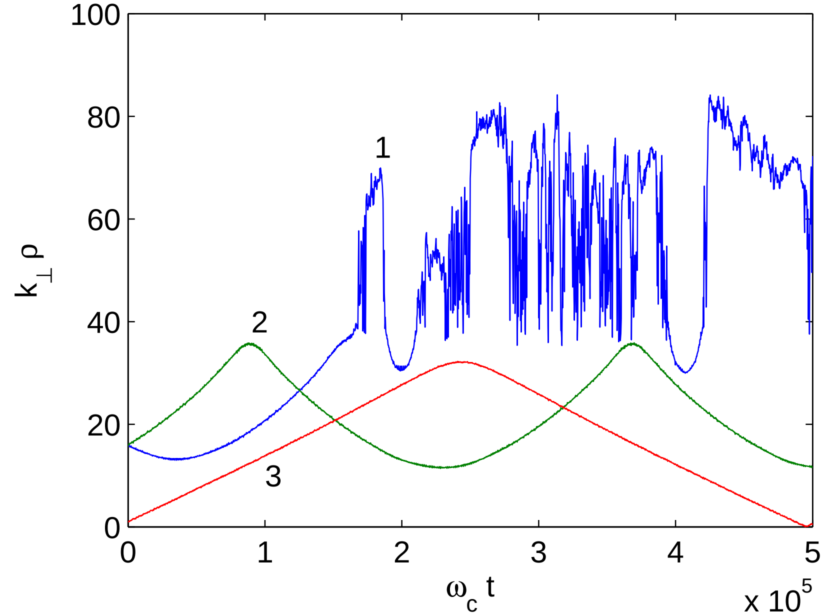
One Perp. Wave: Stochastic Region

$\omega = 39.37\omega_c$



Two Perp. Waves: Coherent Motion

$\omega_1 = 40.37\omega_c, \ \omega_2 = 39.37\omega_c$



- Coherent Motion: Analytic Perturbation Theory
- $\omega_1 - \omega_2 = N\omega_c$
  - Perpendicular Waves
  - Oblique Waves
  - Coherent Motion in Multiple Discrete Waves
  - Continuous Spectrum $\rightarrow$ time-varying amplitudes
- $\omega_1 + \omega_2 = N\omega_c$: Heating below ω_c for $N = 1, 2$

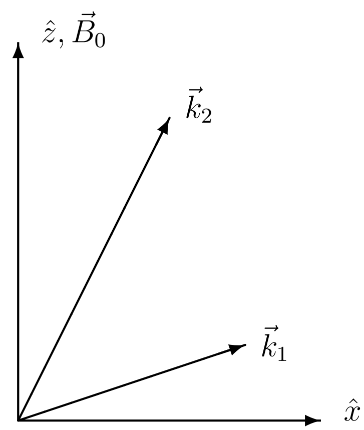
Equations of Motion

- Motion in two electrostatic waves and uniform $\vec{B} = B_0\hat{z}$:

$$M\frac{d^2\vec{x}}{dt^2} = q\sum_{i=1}^{2}\Phi_i\vec{k}_i\sin(\vec{k}_i\cdot\vec{x} - \omega_i t + \alpha_i) + q\vec{v}\times\vec{B}$$

- Normalizations: time to $\omega_c \equiv qB_0/M$, distances to $k_{1\perp}$.

$$\nu_i \equiv \frac{\omega_i}{\omega_c} \qquad \epsilon_i \equiv \left(\frac{\omega_{Bi}}{\omega_c}\right)^2 \qquad \omega_{Bi} \equiv \sqrt{\frac{qk_{1\perp}^2\Phi_i}{M}}$$



$$k_x = k_\perp$$
$$k_y = 0$$
$$k_z = k_\parallel$$

- Hamiltonian formulation:

$$h(\vec{p},\vec{x},t) = \tfrac{1}{2}(\vec{p} - x\hat{y})^2 + \sum_i \epsilon_i\cos(k_{i\perp}x + k_{i\parallel}z - \nu_i t + \alpha_i)$$

- Gyro-Variables:

$$\rho \equiv \sqrt{v_x^2 + v_y^2} = \text{gyroradius} \qquad I \equiv \frac{\rho^2}{2} = \text{perp. energy} \qquad \phi \equiv \arctan\left(-\frac{v_y}{v_x}\right) = \text{gyrophase}$$

- Hamiltonian for gyro-variables:

$$H(I, v_z, \phi, z, t) = I + \tfrac{1}{2}v_z^2 + \sum_i \epsilon_i\cos(k_{i\perp}\rho\sin\phi + k_{i\parallel}z - \nu_i t + \alpha_i)$$

Lie Perturbation Theory for Multiple Waves

[A. Lichtenberg, M. Lieberman, "Regular and Stochastic Motion" (1992), J.Cary, *Physics Reports* 79,2 (1981)]

- Physical variables: ϕ, z, I, v_z with Hamiltonian H

$$H = H_0 + H_1 \qquad H_0 = I + \tfrac{1}{2}v_z^2 \qquad H_1 = \sum_i \epsilon_i \cos(k_{i\perp}\rho \sin\phi + k_{i\parallel}z - \nu_i t + \alpha_i)$$

- Lie-transform variables: $\bar{\phi}, \bar{z}, \bar{I}, \bar{v}_z$ with coherent Hamiltonian $\bar{H}$

$$\bar{H} = \bar{H}_0 + \bar{H}_1 + \bar{H}_2 \qquad \bar{H}_0 = \bar{I} + \tfrac{1}{2}\bar{v}_z^2$$

- $O(\epsilon)$: No resonant terms unless $\nu_i \in \mathcal{Z}$

$$\boxed{\bar{H}_1 = 0}$$

- $O(\epsilon^2)$: Second-order resonance if $\nu_1 - \nu_2 \in \mathcal{Z}$, $\nu_1 + \nu_2 \in \mathcal{Z}$, $2\nu_i \in \mathcal{Z}$.

$$\begin{aligned}\bar{H}_2 = S_0(I, v_z) &+ \delta_- S_-(I, v_z)\cos((\nu_1 - \nu_2)(\phi - t) + (k_{1\parallel} - k_{2\parallel})z + \alpha_1 - \alpha_2) \\ &+ \delta_+ S_+(I, v_z)\cos((\nu_1 + \nu_2)(\phi - t) + (k_{1\parallel} + k_{2\parallel})z + \alpha_1 + \alpha_2) \\ &+ \sum_i \delta_i S_i(I, v_z)\cos(2\nu_i(\phi - t) + 2k_{i\parallel}z + 2\alpha_i)\end{aligned}$$

$\delta_-, \delta_+, \delta_i = 1$ when $\nu_1 - \nu_2,\ \nu_1 + \nu_2,\ 2\nu_i \in \mathcal{Z}, 0$ otherwise.

- Rotating gyro-frame:

$$\boxed{\bar{\psi} = \bar{\phi} - t \qquad \Longrightarrow \qquad \bar{H}(\bar{\psi}, \bar{z}, \bar{I}, \bar{v}_z) = \tfrac{1}{2}\bar{v}_z^2 + \bar{H}_2}$$

Expression for S's

$$S_0 = -\frac{1}{2\bar{\rho}}\sum_{i,m} k_{i\perp}\epsilon_i^2 \frac{m}{m-\mu_i} J_{m,i}J'_{m,i} + \tfrac{1}{4}\sum_{i,m} k_{i\parallel}^2 \epsilon_i^2 \frac{J_{m,i}^2}{(m-\mu_i)^2}$$

$$S_- = \frac{\epsilon_1\epsilon_2}{4}\sum_m \left\{ \frac{-1}{\bar{\rho}(m-\mu_1)}(k_{1\perp}(m-N_-)J'_{m,1}J_{m-N_-,2} + k_{2\perp} m J_{m,1} J'_{m-N_-,2}) \right.$$

$$\left. + k_{1\parallel}k_{2\parallel}\frac{J_{m,1}J_{m-N_-,2}}{(m-\mu_1)^2} + \text{the same with subscripts 1 and 2 switched, } N_- \rightarrow -N_- \right\}$$

$$S_+ = -\frac{\epsilon_1\epsilon_2}{4}\sum_m \left\{ \frac{1}{\rho(m-\mu_1)}(k_{1\perp}(m-N_+)J'_{m,1}J_{-m+N_+,2} + k_{2\perp} m J_{m,1} J'_{-m+N_+,2}) \right.$$

$$\left. + k_{1\parallel}k_{2\parallel}\frac{J_{m,1}J_{-m+N_+,2}}{(m-\mu_1)^2} + \text{the same with subscripts 1 and 2 switched} \right\}$$

$$S_i = -\frac{\epsilon_i^2}{4}\sum_m \left\{ \frac{1}{\rho(m-\mu_i)}(m k_{i\perp} J_{m,i} J'_{-m+2\nu_i,i} + (m-2\nu_i)k_{i\perp} J'_{m,i} J_{-m+2\nu_i,i}) \right.$$

$$\left. + k_{i\parallel}^2 \frac{J_{m,i}J_{-m+2\nu_i,i}}{(m-\mu_i)^2} \right\}$$

$$\mu_i = \nu_i - k_{i\parallel}v_z \qquad J_{m,i} = J_m(k_{i\perp}\bar{\rho}) \qquad N_- = \nu_1 - \nu_2 \qquad N_+ = \nu_1 + \nu_2$$

One Second-Order Resonance: $\omega_1 - \omega_2 = N\omega_c$

$$\bar{H} = \tfrac{1}{2}\bar{v}_z^2 + S_0(\bar{I}, \bar{v}_z) + S_-(\bar{I}, \bar{v}_z)\cos(N\bar{\psi} - \Delta k_\parallel \bar{z})$$

- Coherent Motion in v_z

$$\frac{d}{dt}\left(\bar{v}_z - \frac{\Delta k_\parallel}{N}\bar{I}\right) = 0 \qquad \Longrightarrow \qquad \boxed{\bar{v}_z(\bar{I}) = v_{z0} + \frac{\Delta k_\parallel}{N}(\bar{I} - I_0)}$$

- Bounds of coherent motion

$$\cos(N\bar{\psi} - \Delta k_\parallel \bar{z}) = \frac{\bar{H} - \frac{1}{2}\bar{v}_z^2 - S_0}{S_-} \qquad \Longrightarrow \qquad |\bar{H} - \tfrac{1}{2}\bar{v}_z^2 - S_0| > |S_-| \qquad \text{forbidden}$$

- Potential barriers:

$$\boxed{H_\pm(\bar{I}) = \tfrac{1}{2}\bar{v}_z^2 + S_0 \pm |S_-| \qquad\qquad H_- \leq \bar{H} \leq H_+}$$

Turning points in $\bar{I}$:

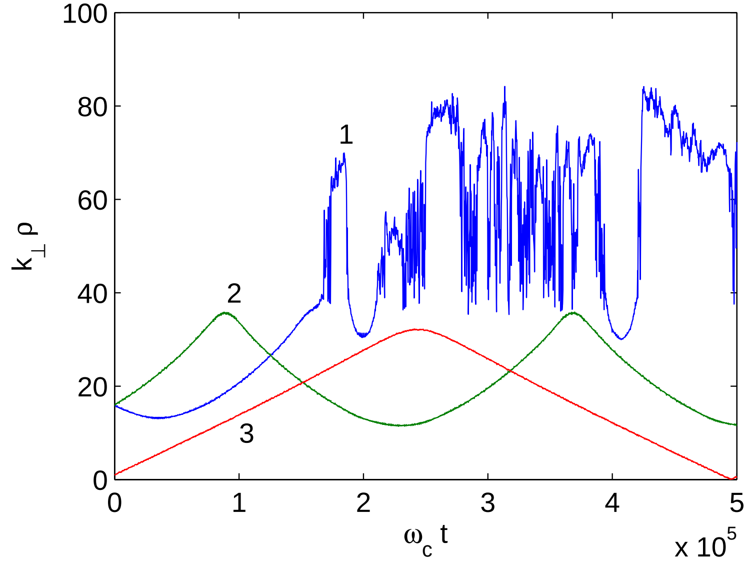
$$\bar{H} = \tfrac{1}{2}\bar{v}_z^2 + S_0 \pm |S_-|$$

Occur when

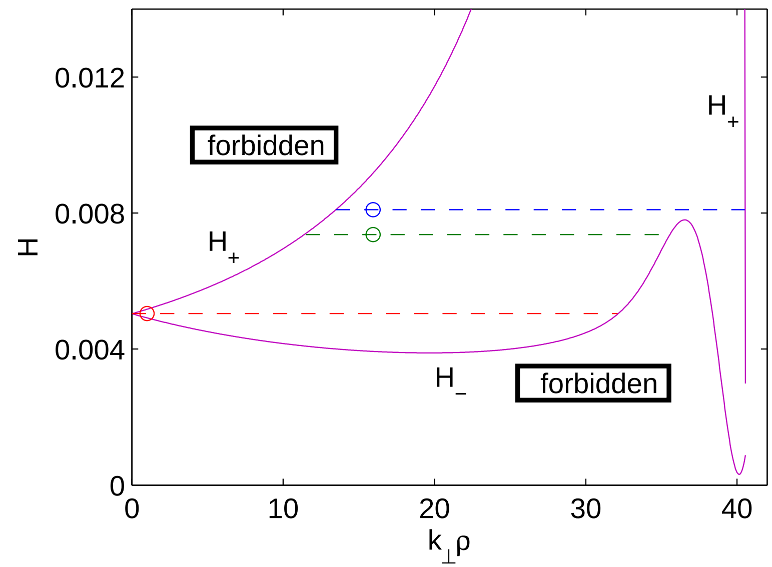
$$N\bar{\psi} - \Delta k_\parallel \bar{z} = m\pi$$

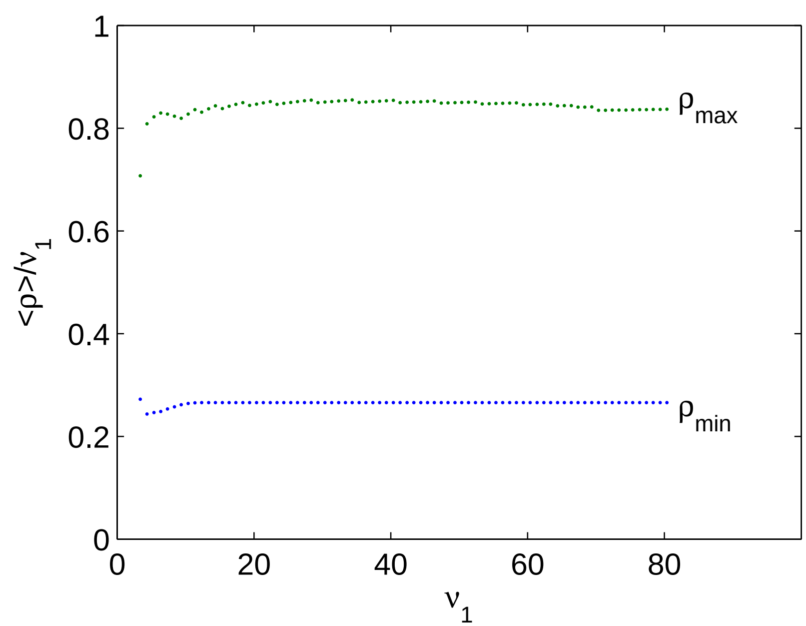
$\omega_1 - \omega_2 = N\omega_c, k_{i\|} = 0$: Perpendicular Energization

$\nu_1 = 40.37, \nu_2 = 39.37, \epsilon_i = 4, k_{i\perp} = 1$



Potential Barriers $H_\pm$





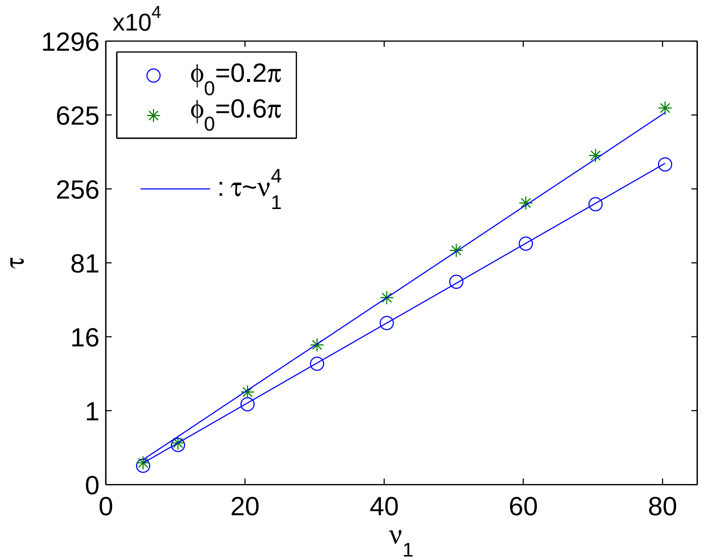
- Range of coherent motion in ρ is linear in ν_1
- $\langle \rho \rangle$ = Average over initial phase ϕ_0 for $\rho_0 = 0.4\nu_1$

Oscillation Period Increases with Wave Frequency

- Period of Coherent Oscillation τ:

$$\tau \equiv \frac{2\pi}{N\left\langle d\bar{\psi}/dt\right\rangle} \sim \frac{\nu_1^4}{N\epsilon_1^2}$$

- Comes from asymptotic forms for S's



$\omega_1 - \omega_2 \neq N\omega_c$: Stricter Limits for Higher Wave Frequencies

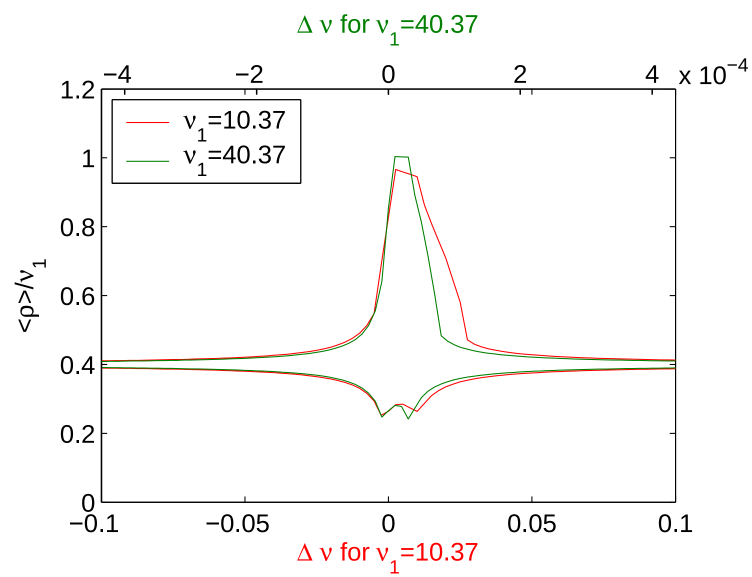
$$\nu_2 = \nu_1 - 1 - \Delta\nu$$

$\nu_1 - \nu_2 = N + \Delta\nu$
$\Delta\nu$ = departure from resonance.

$\rho(\Delta\nu)$ = min. or max. ρ of orbit

ρ_a, ρ_b for frequencies $\{\nu_{1a}, \nu_{2a}\}$, $\{\nu_{1b}, \nu_{2b}\}$

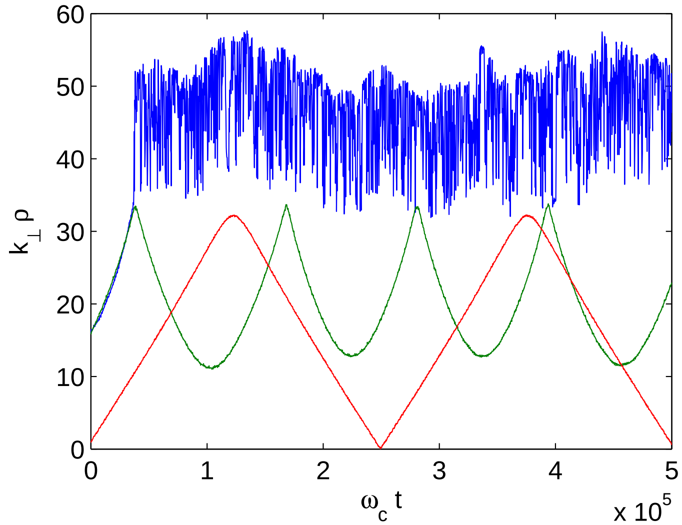
$$\rho_b(\Delta\nu) \approx \frac{\nu_{1b}}{\nu_{ia}}\rho_a\left(\left(\frac{\nu_{1b}}{\nu_{1a}}\right)^4 \Delta\nu\right)$$

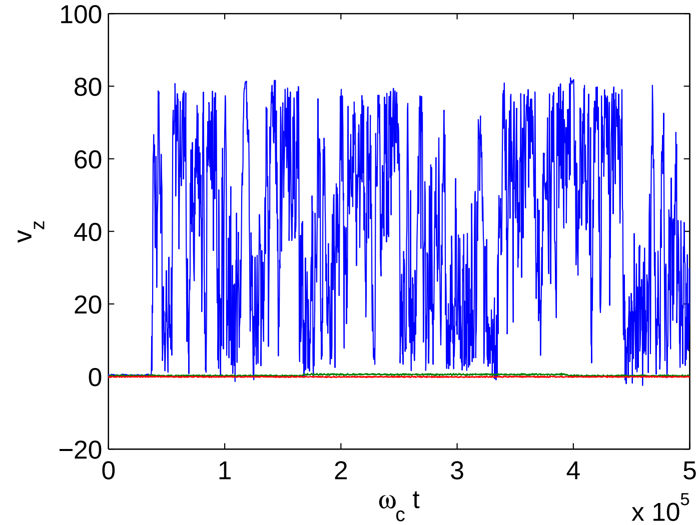


$\omega_1 - \omega_2 = N\omega_c,\ k_{1\|} = k_{2\|}$: Coherent Motion in ρ

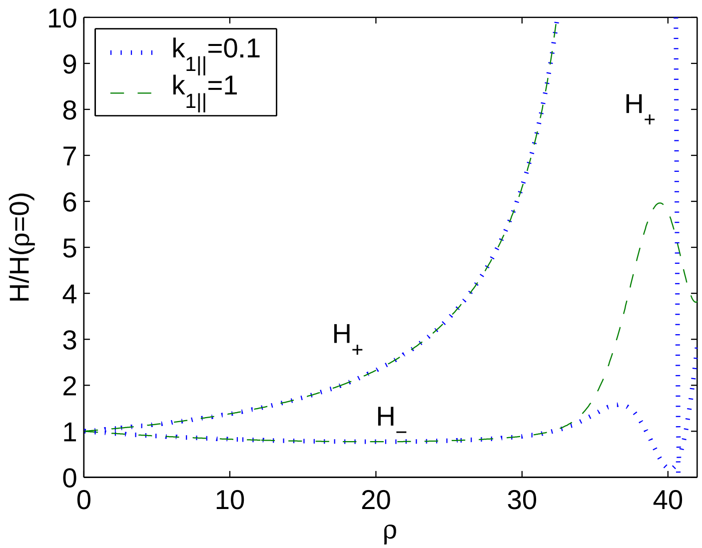
$\bar{v}_z = v_{z0} + \frac{\Delta k_\|}{N}(\bar{I} - I_0)$ $\Delta k_\| = 0$: no parallel coherent motion

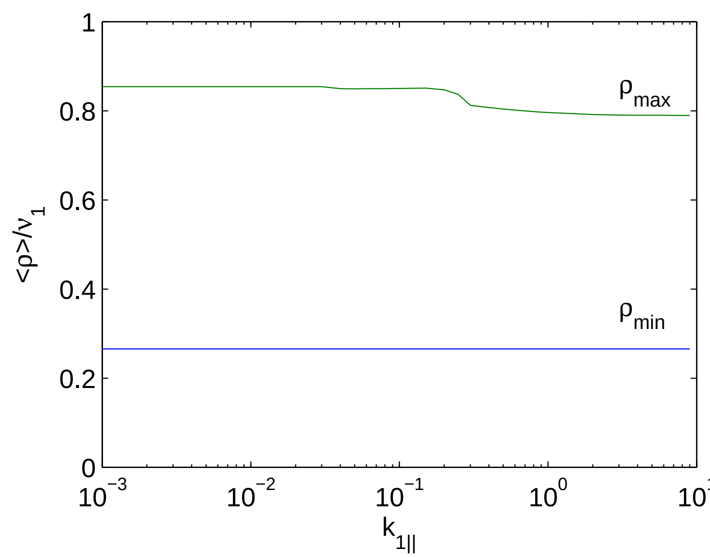
- 45° waves: $\nu_1 = 40.37,\ \nu_2 = 39.37,\ \epsilon_i = 4,\ k_{i\perp} = k_{i\|} = 1$





- Arbitrary angle: Range of motion in ρ Changes Slightly

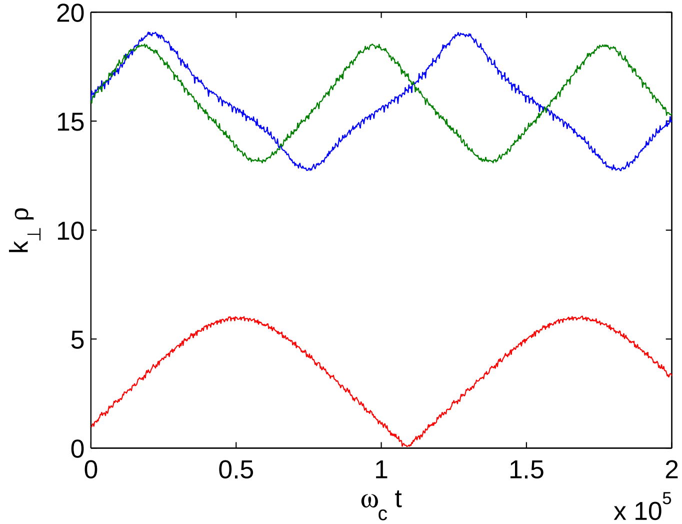


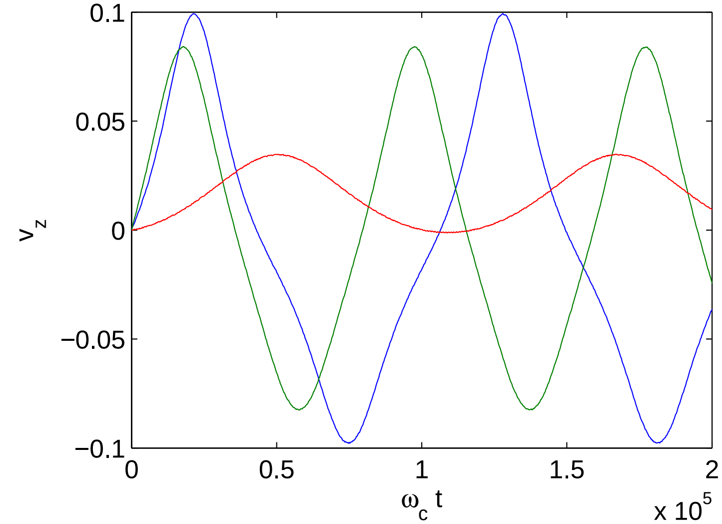


$\omega_1 - \omega_2 = N\omega_c,\ k_{1\|} \neq k_{2\|}$: Coherent Motion Limited

$\bar{v}_z = v_{z0} + \frac{\Delta k_\|}{N}(\bar{I} - I_0)$ $\Delta k_\| \neq 0$: small parallel coherent motion, detunes waves

- $\nu_1 = 40.37,\ \nu_2 = 39.37,\ \epsilon_i = 4,\ k_{i\perp} = 1,\ k_{1\|} = 0.002,\ k_{2\|} = 0$



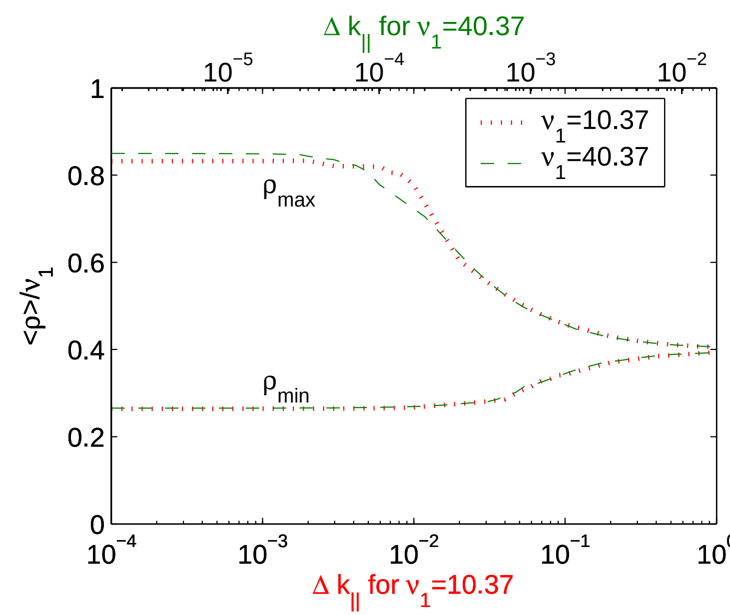


$$\bar{H} = \frac{1}{2}\underbrace{\frac{\Delta k_\|^2}{N^2}(\bar{I} - \bar{I}_0)^2}_{\bar{v}_z^2} + S_0 + S_- \cos(N\bar{\psi} - \Delta k_\| \bar{z})$$

$\rho(\Delta k_\|) =$ Min. or max. ρ of orbit

ρ_a, ρ_b for frequencies $\{\nu_{1a}, \nu_{2a}\},\ \{\nu_{1a}, \nu_{2a}\}$

$$\rho_b(\Delta k_\|) \approx \frac{\nu_{1b}}{\nu_{ia}}\rho_a\left(\left(\frac{\nu_{1b}}{\nu_{1a}}\right)^3 \Delta k_\|\right)$$



Continuous Spectrum: Change in Wave Amplitudes

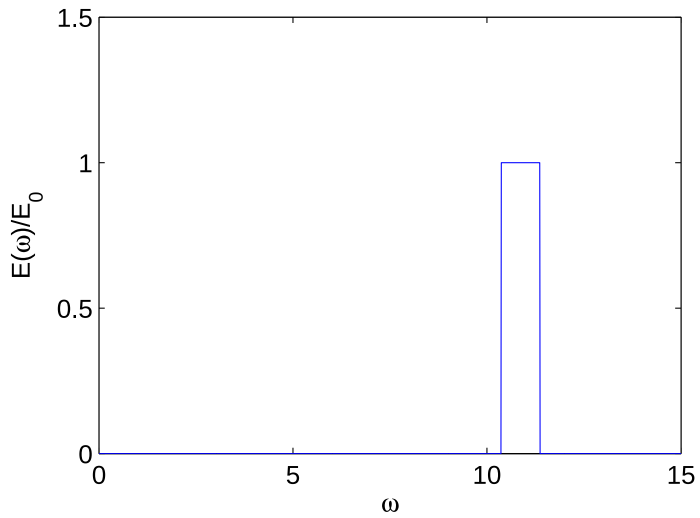
$$E(x,t) = \int d\omega \hat{E}(\omega) \cos(k(\omega)x - \omega t + \alpha(\omega))$$

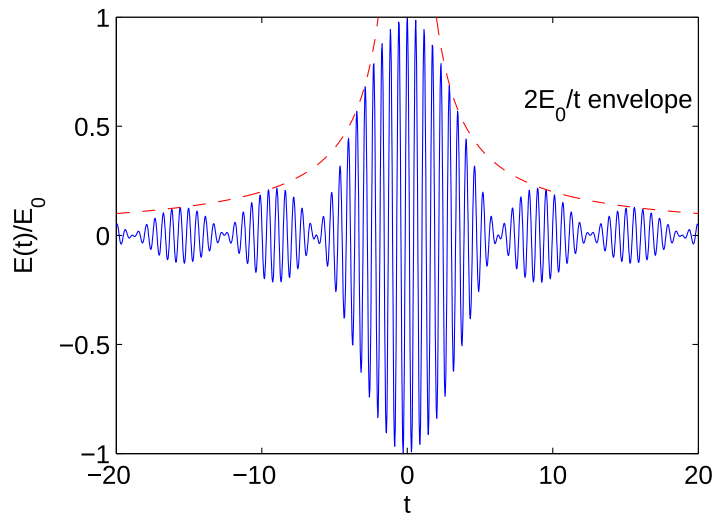
- Box spectrum: $k(\omega) = 1 \qquad \alpha(\omega) = 0 \qquad \hat{E}(\omega) = \begin{cases} E_0 & \omega_1 < \omega < \omega_2 \\ 0 & \text{otherwise} \end{cases}$

$$E = \frac{E_0}{t}(\sin(x - \omega_1 t) - \sin(x - \omega_2 t))$$

Same field as two discrete waves with $1/t$ amplitude dropoff.

- Example: $\omega_1 = 10.37, \omega_2 = 11.37$

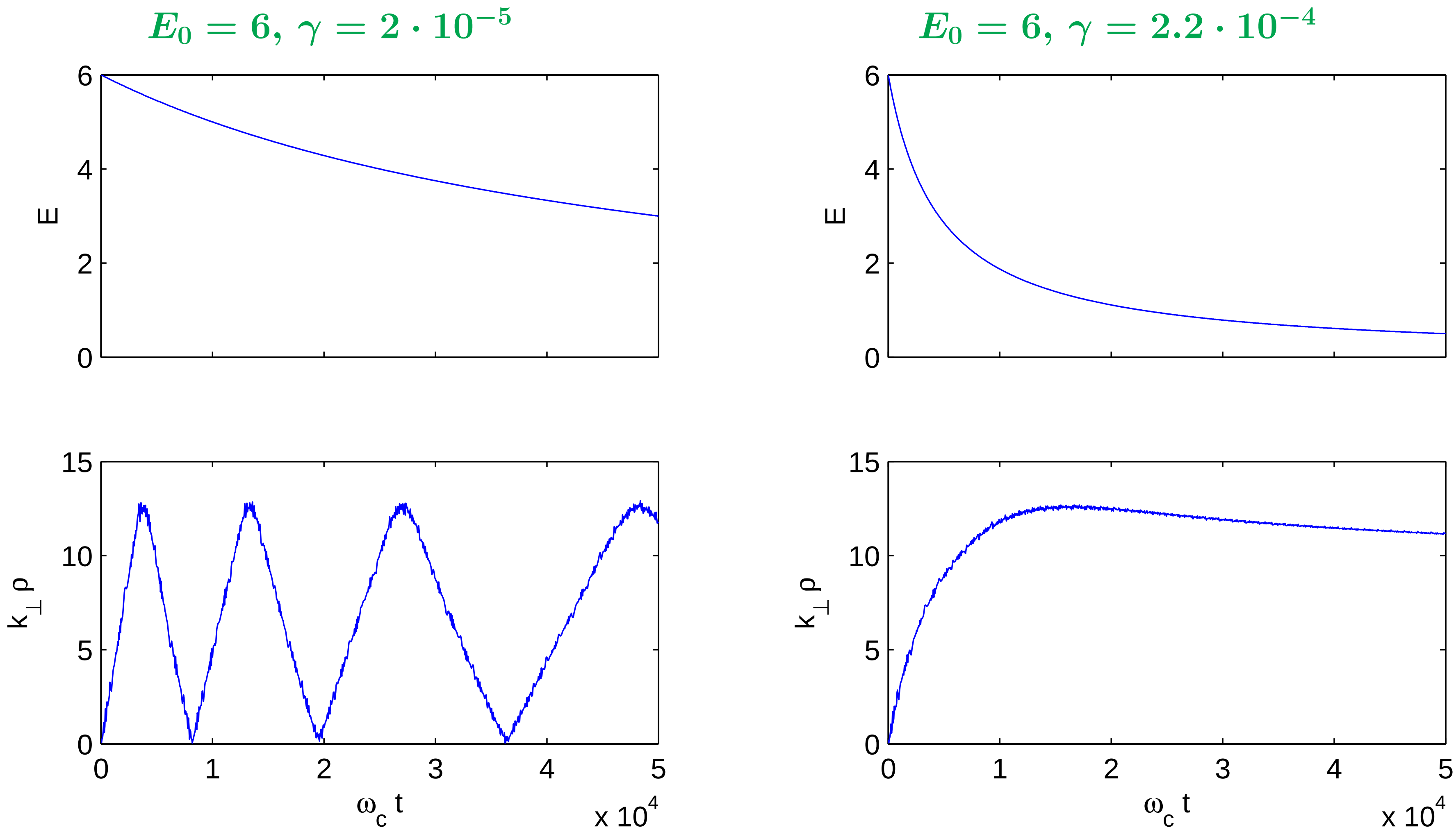




- Shift t: $t \rightarrow t + t_0 \qquad E = \frac{\epsilon}{1+\gamma t}(\sin(x - \omega_1 t) - \sin(x - \omega_2 t)) \qquad \epsilon = \frac{E_0}{t_0},\ \gamma = \frac{1}{t_0}$

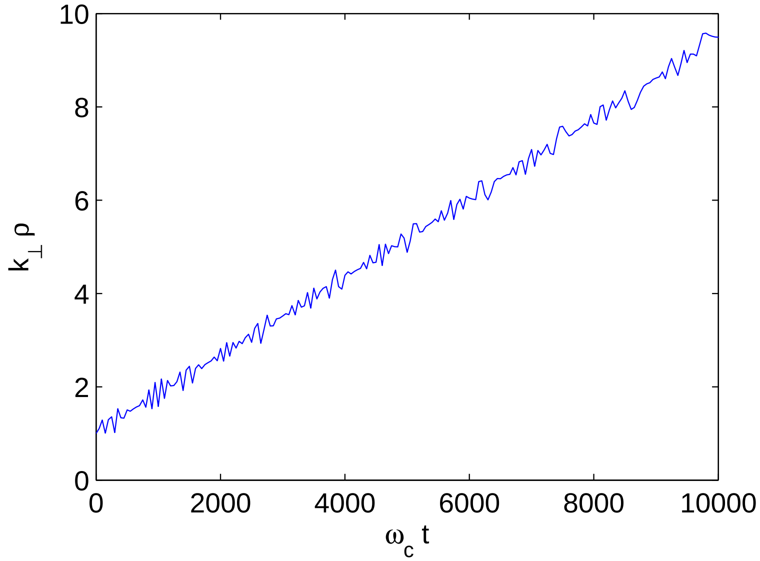
Changing Amplitude: Rate of Energization Changes

- $\omega_1 = 16.43\omega_c,\ \omega_2 = 17.43\omega_c$

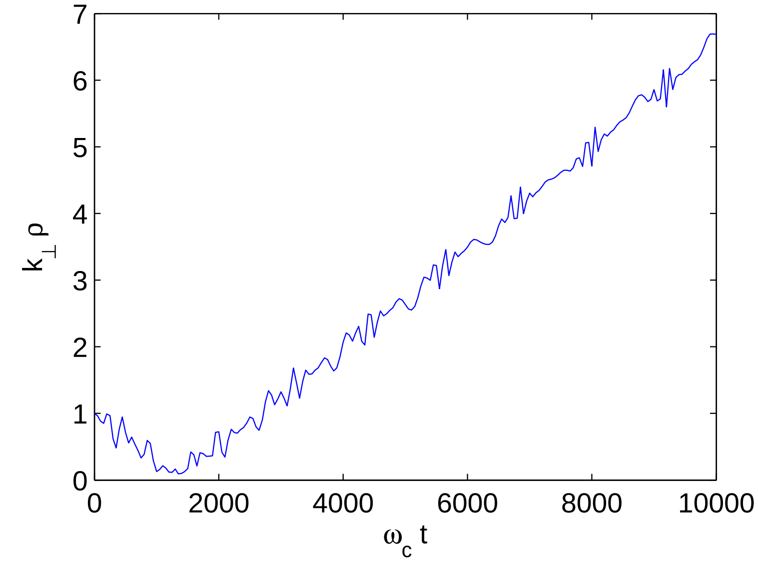


Multiple Waves: Energization If Phases Match

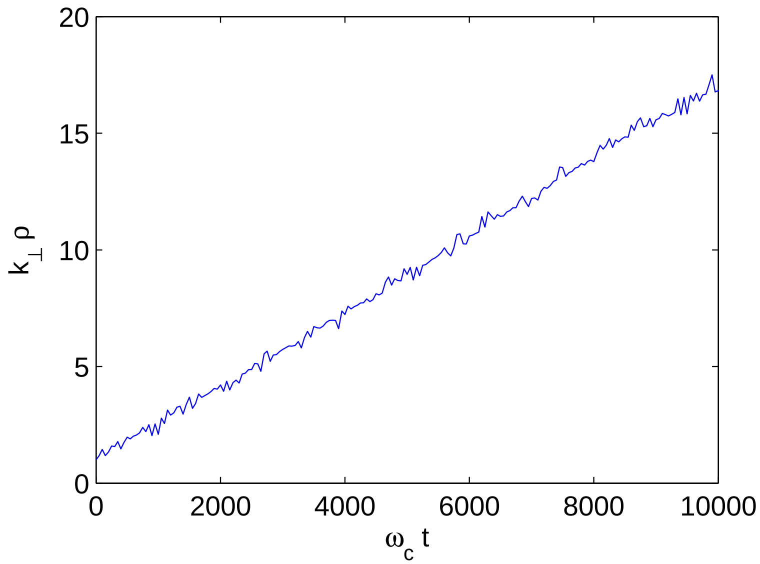
$\nu_1 = 21.81,\ \nu_2 = 20.81,\ \vec{\alpha} = (0, 0)$



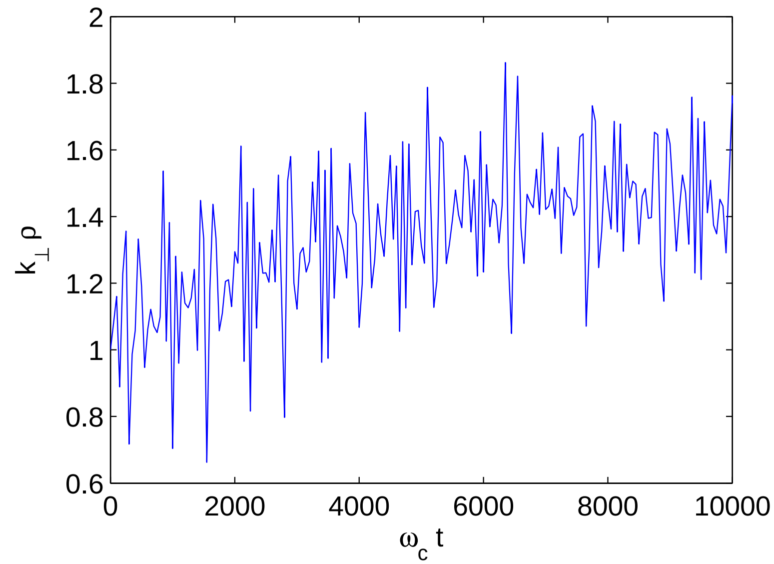
$\nu_1 = 22.64,\ \nu_2 = 21.64,\ \vec{\alpha} = (\pi, 0)$



4 waves, $\vec{\alpha} = (0, 0, 0, 0)$

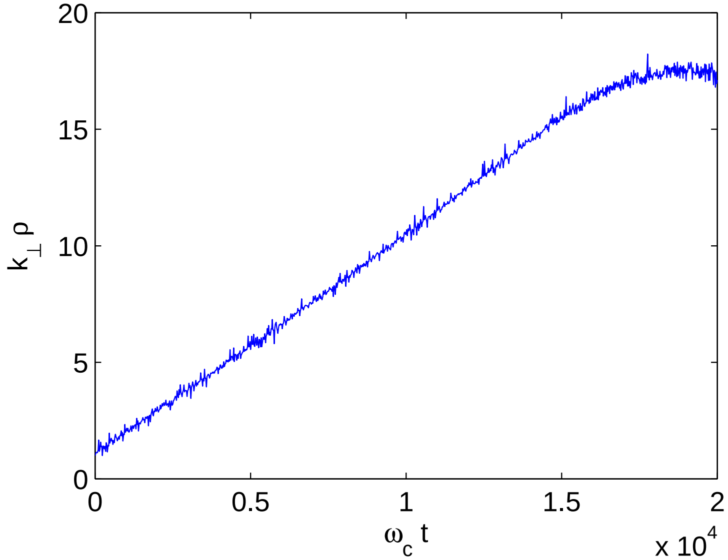


4 waves, $\vec{\alpha} = (0, 0, \pi, 0)$

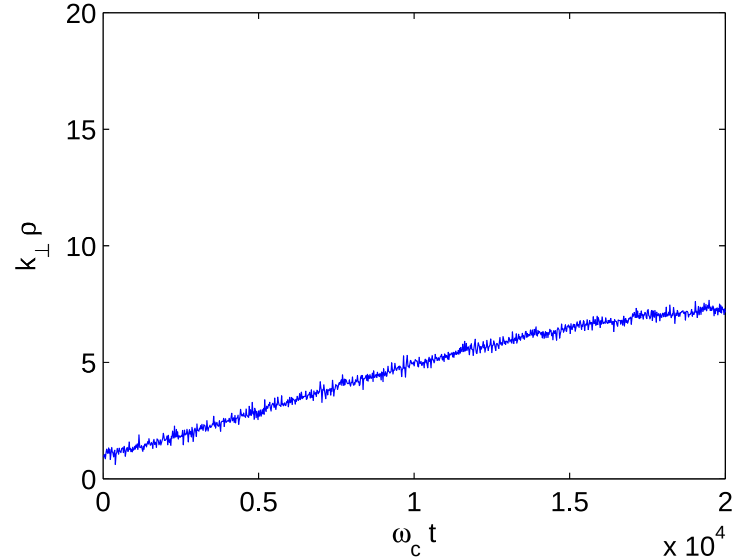


Multiple Waves: Energization If Enough ω's Match

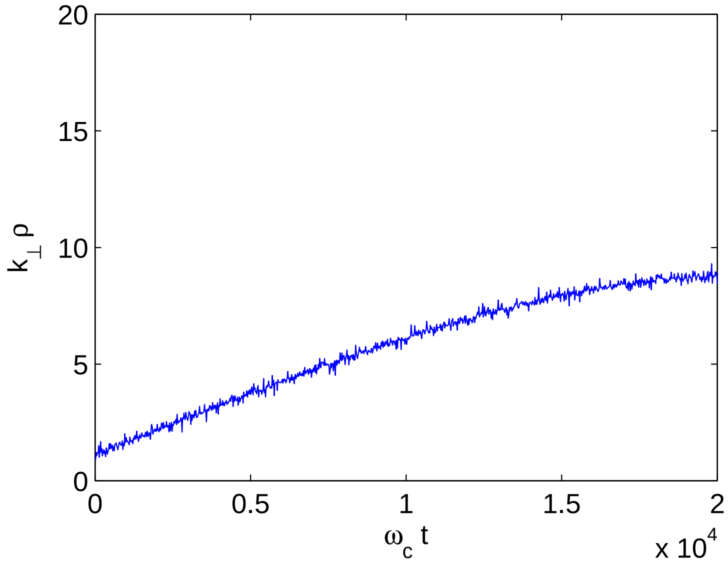
10 waves, $\nu_i - \nu_{i+5} = 1, \ \vec{\alpha} = 0$



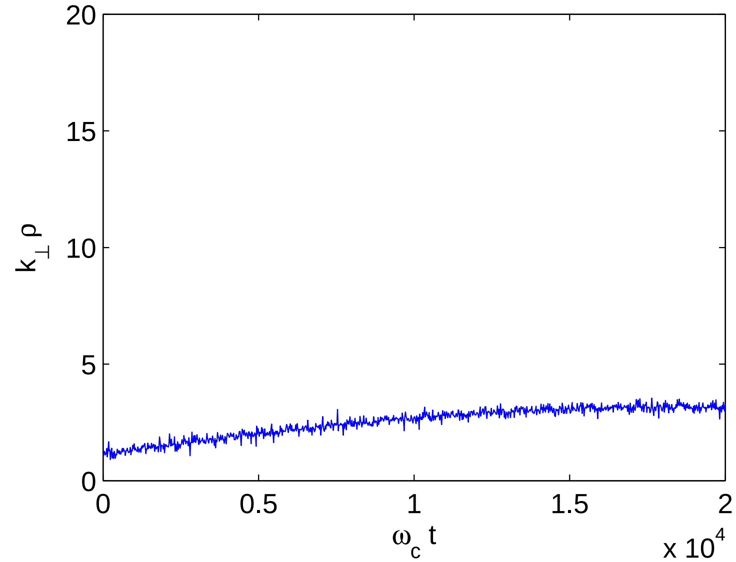
$\nu_i - \nu_{i+5} = 1, \ \vec{\alpha} = \text{random}$



$\nu_i - \nu_{i+5} = 1$ for 6 of 10 waves, $\vec{\alpha} = 0$



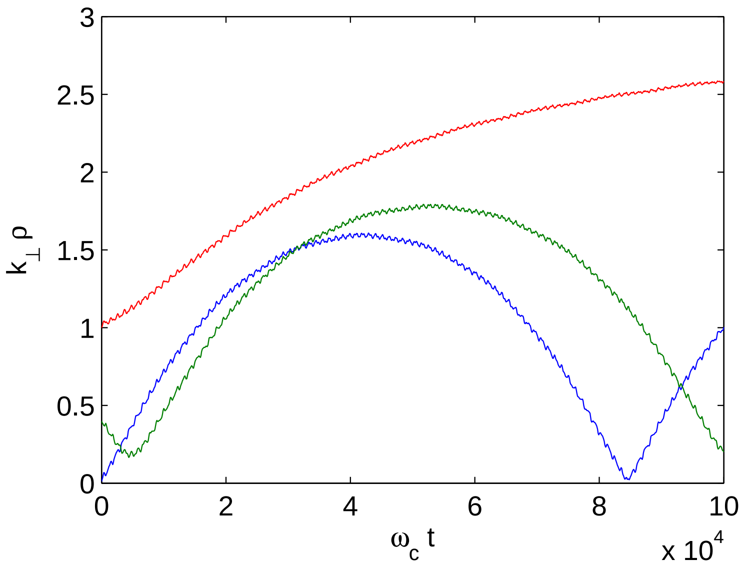
$\nu_i - \nu_{i+5} = 1$ for 2 of 10 waves, $\vec{\alpha} = 0$



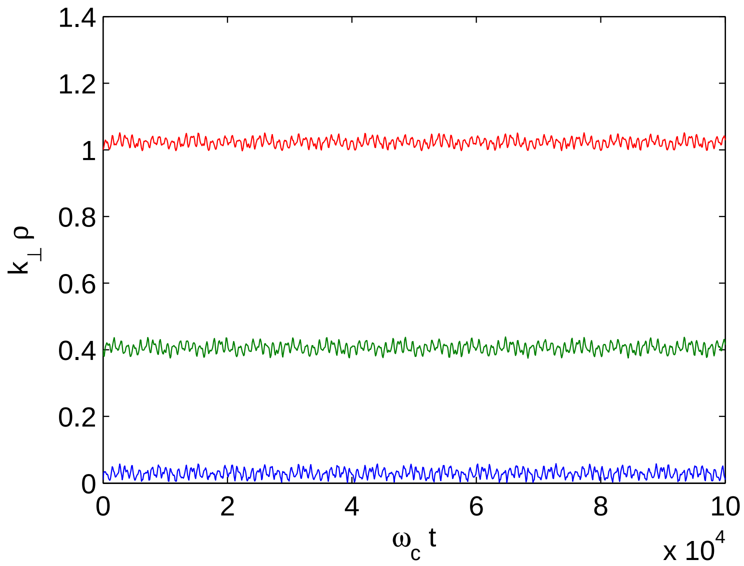
$\omega_1 + \omega_2 = N\omega_c$: Coherent Energization for $N = 1, 2$

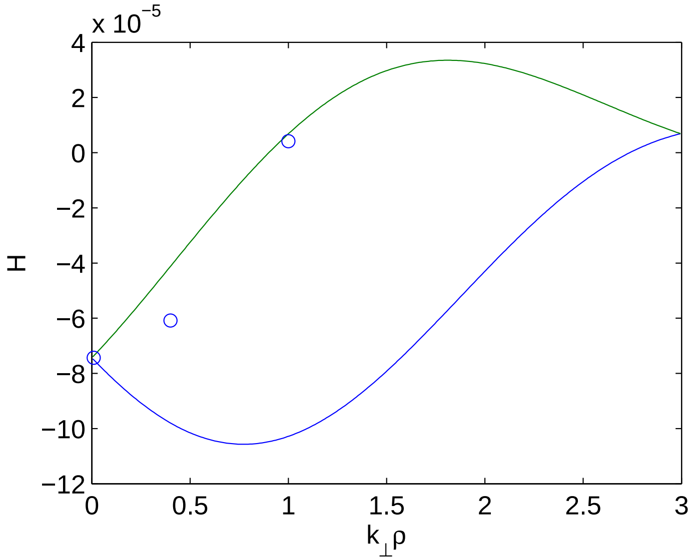
- Can energize with waves below ω_c

$\nu_1 = 0.32, \nu_2 = 0.68, \epsilon_i = 0.01$



$\nu_1 = 0.31, \nu_2 = 0.68, \epsilon_i = 0.01$





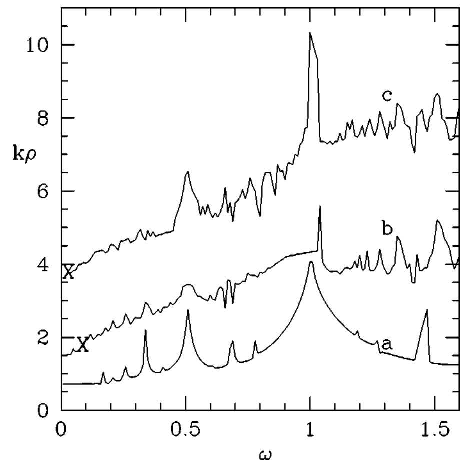
Potential barriers $H_\pm$

$$\bar{H} = S_0 + S_+ \cos(N\bar{\psi})$$

$$H_- \leq \bar{H} \leq H_+ \qquad H_\pm(\bar{I}) \equiv S_0(\bar{I}) \pm \left|S_+(\bar{I})\right|$$

Energization Comparable to Stochastic Heating by Large-Amplitude Wave

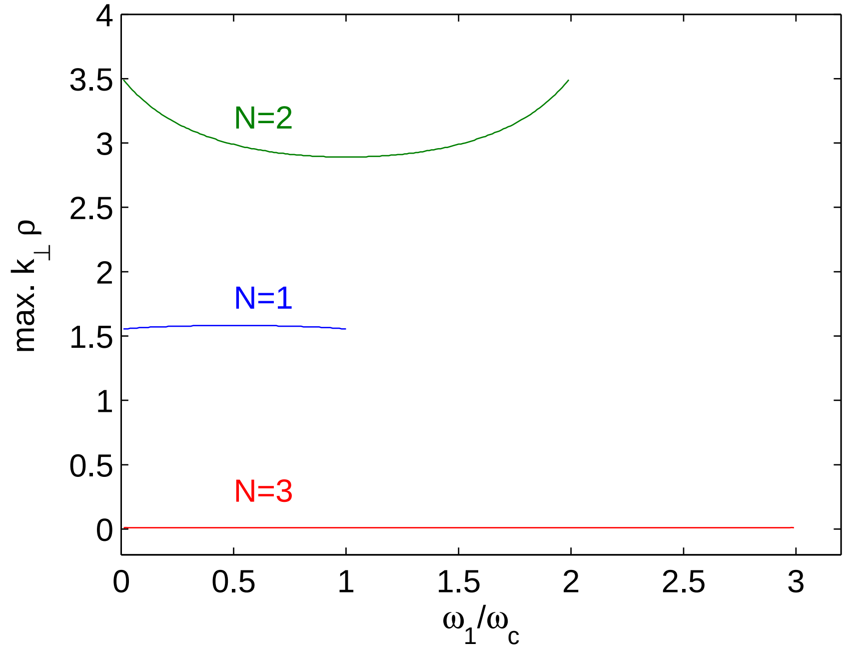
Stochastic Heating

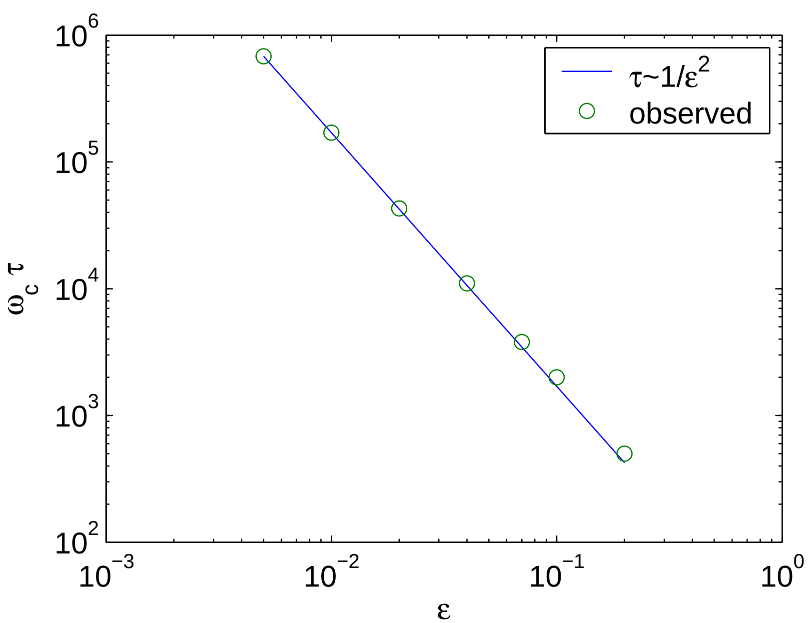


Heating domain vs. ω/ω_c for $\epsilon =$ (a) 0.36, (b) 0.8, (c) 2.6

From L. Chen, Z. Lin, R. White, *Phys. Plasmas* 8,11 (2001)

Coherent Energization ($\rho_0 = 0.01$)





$\tau =$ time for particle to reach first max.

$$\tau \sim \frac{1}{\epsilon^2}$$

- Figure: $\rho_0 = 0.01,\ \nu_1 = 0.32,\ \nu_2 = 0.68,\ \epsilon_i = \epsilon$

Summary

- Coherent Energization of low-energy particles by two waves when $\omega_1 - \omega_2 \in \mathcal{Z}, \quad \omega_1 + \omega_2 \in \mathcal{Z}$.
- Lie Perturbation Theory explains the coherent motion, elucidates scalings with wave parameters.
- No minimum wave amplitude for energization to occur.
- Timescale for energization scales as $1/\epsilon^2$ and $(\omega/\omega_c)^4$.
- $\omega_1 - \omega_2 = N\omega_c$
  - Oblique waves with $k_{1\|} = k_{2\|}$ can increase particle's perpendicular but not parallel energy.
  - Deviations from $\omega_1 - \omega_2 = N\omega_c$ and $k_{1\|} = k_{2\|}$ for which energization occurs are larger for lower wave frequencies.
  - Variation in wave amplitudes (e.g., due to continuous frequency spectrum) changes the rate of energization but does not destroy the process.
  - Multiple waves: coherent energization can "add", depending on whether wave phases match and what fraction of waves are non-resonant.
- $\omega_1 + \omega_2 = N\omega_c$
  - Coherent energization if $N = 1, 2$.
  - Amplitudes needed are much smaller than those for stochastic heating by one wave below ω_c.

Comments? Reprint? Please leave address.