

Cone-Guided Fast Ignition with Imposed Magnetic Fields

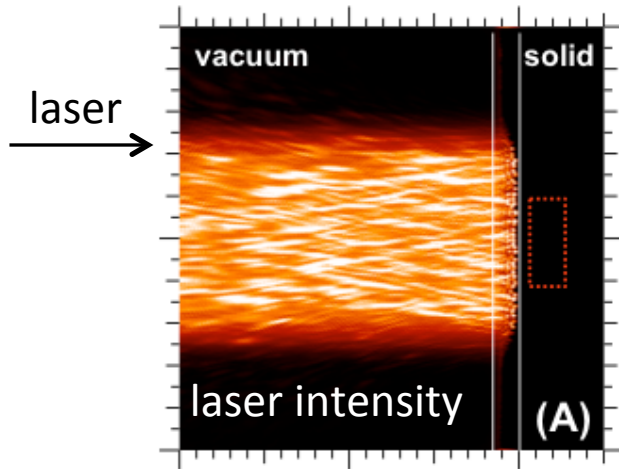
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M. H. Key, L. Divol, A. J. Kemp, H. D. Shay**
LLNL

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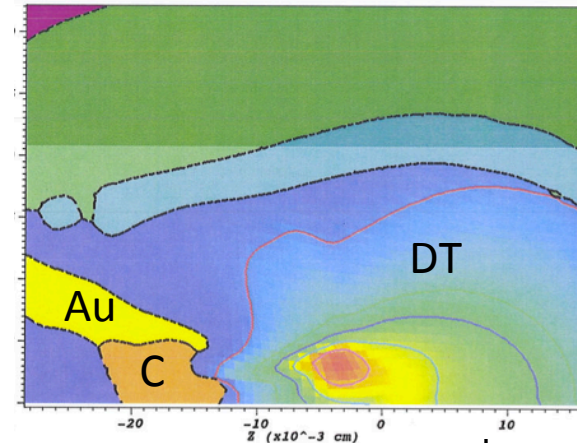
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Release num. LLNL-CONF-488273

Our fast ignition modeling approach

Explicit PIC modeling of short-pulse laser-plasma interaction: A. J. Kemp, L. Divol

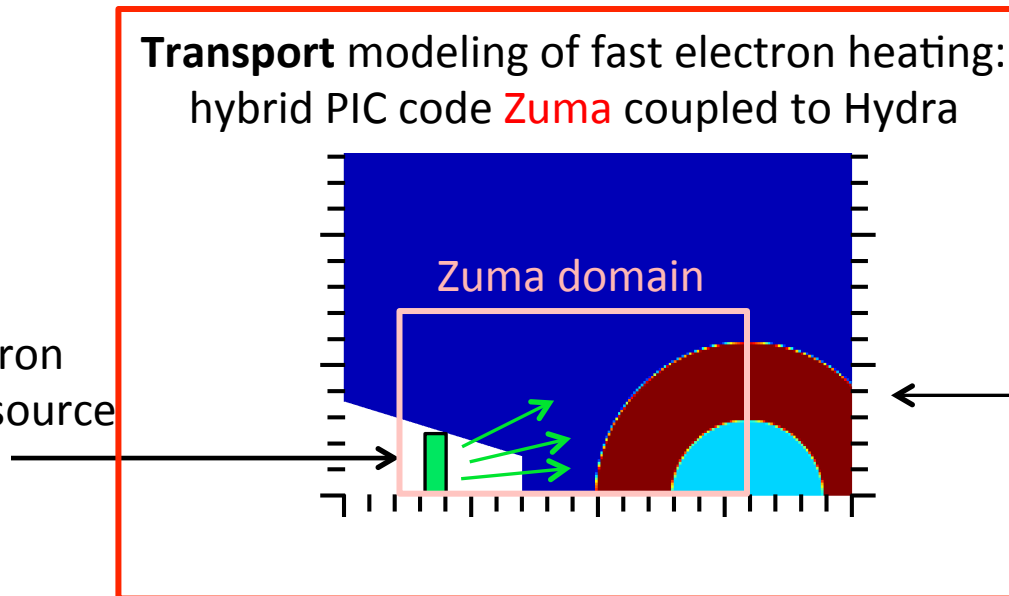


Rad-hydro: fuel assembly in hohlraum, around cone: H. D. Shay, M. Tabak, D. Ho



Transport modeling of fast electron heating:
hybrid PIC code **Zuma** coupled to Hydra

fast electron
injected source



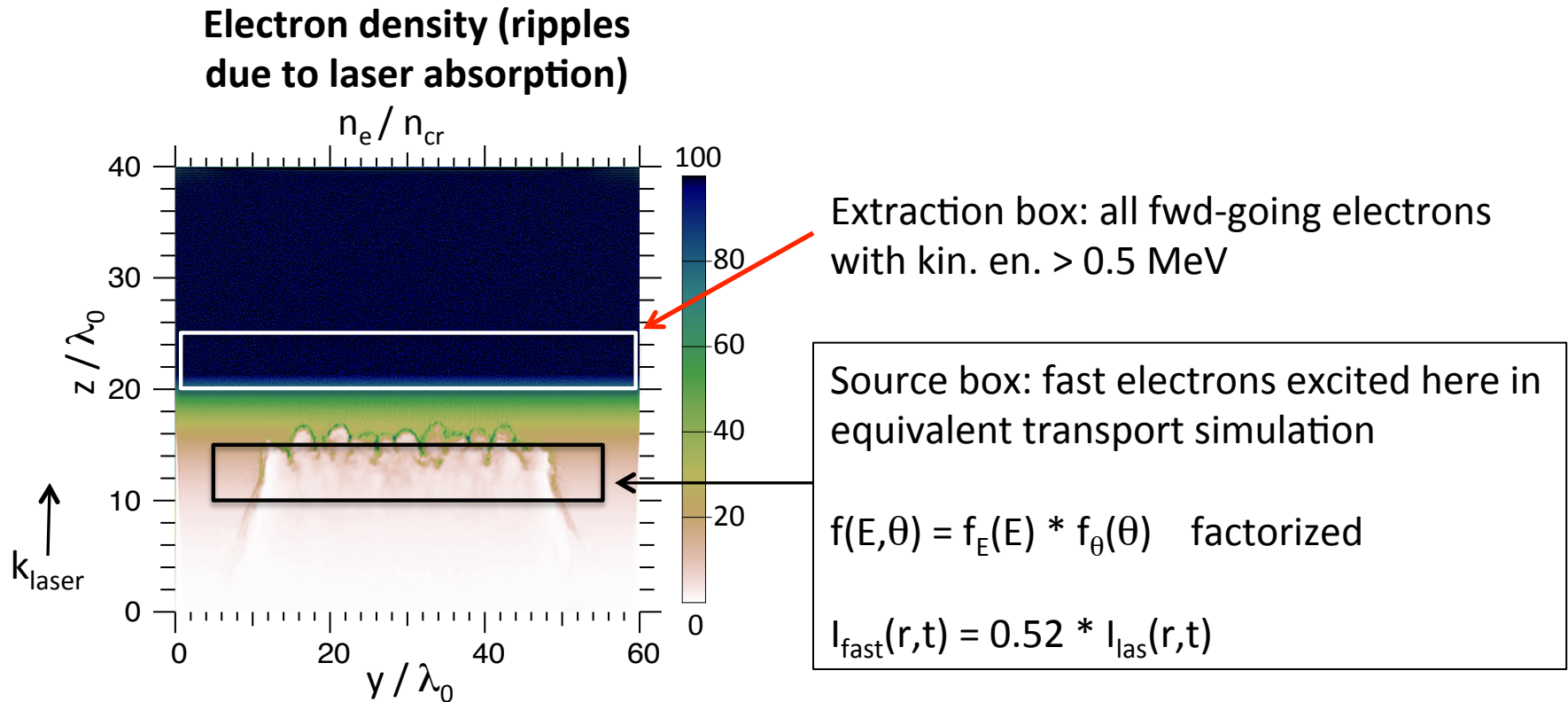
plasma conditions
at time of ignitor pulse

Subject of this talk

Some trick, like imposed magnetic fields, is needed to achieve fast ignition with a realistic, divergent fast electron source

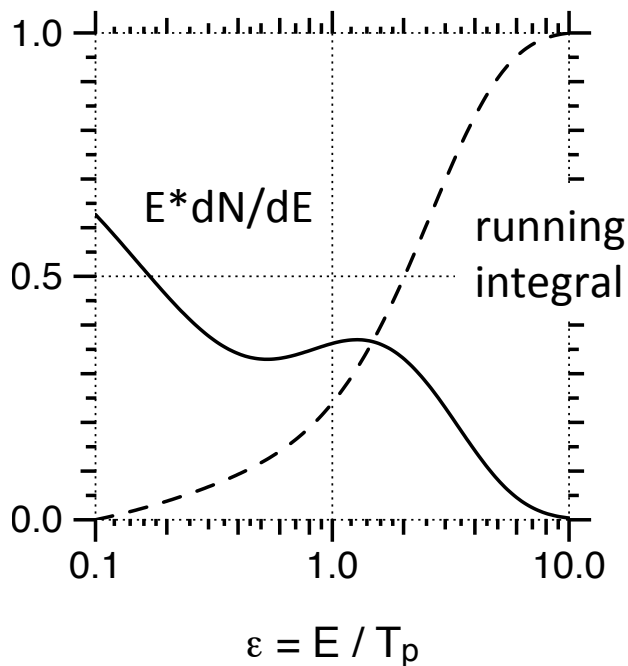
- **Fast electron source** generated by short-pulse laser – characterized by PIC sims:
 - **Energy spectrum** has two temperature components, many electrons too energetic to stop in DT hotspot
 - **Angle spectrum** is divergent – serious challenge!
- **Transport modeling:** hybrid PIC code Zuma coupled to Hydra rad-hydro code
- **Imposed uniform axial magnetic fields** > 30 MG mitigate divergence
 - Can be produced in an implosion with seed field ~ 50 kG
- **Magnetic mirroring** in non-uniform field prevents fast electrons from reaching fuel
- **Hollow magnetic pipe** can prevent mirroring

Fast electron source distribution found from explicit PIC laser-plasma simulations with PSC code (A. Kemp, L. Divol)



- 3D Cartesian run, 1 μm laser wavelength, pre-plasma with $n_e \sim \exp[z / 3.5 \mu\text{m}]$
- Intensity at vacuum focus ($z = 10 \mu\text{m}$): $I_{las}(r) = I_0 \exp[-(r/18.3 \mu\text{m})^8]$
- $I_0 = 1.37 \text{ E}20 \text{ W/cm}^2$

PIC fast electron energy spectrum is quasi two-temperature



$$\frac{dN}{d\varepsilon} = \frac{1}{\varepsilon} \exp[-\varepsilon / 0.19] + 0.82 \exp[-\varepsilon / 1.3] \quad \varepsilon = \frac{E}{T_p}$$

We scale dN/dE with ponderomotive temperature [S. C. Wilks et al., Phys. Rev. Lett. (1992)]

$$\frac{T_{\text{pond}}}{m_e c^2} \equiv [1 + a_0^2]^{1/2} - 1 \sim a_0 \equiv \text{sqrt} \left[\frac{I_{\text{las}} \lambda^2}{1.37 \cdot 10^{18} \text{ W cm}^{-2} \mu\text{m}^2} \right]$$

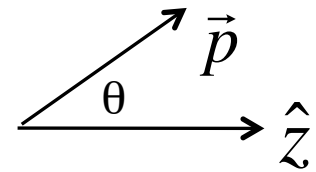
$$\lambda = 1 \mu\text{m}, I_0 = 1.37 \text{ E}20 \text{ W/cm}^2 \rightarrow T_{\text{pond}} = 4.63 \text{ MeV}$$

- Same spectrum in source and extraction box
- Ignition DT hot spot: $\rho \Delta z \sim 1.2 \text{ g/cm}^2$. Removes $\sim 1.4 \text{ MeV}$ from a fast electron (neglecting angular scatter)
 - Spectrum is too energetic to stop in hot spot

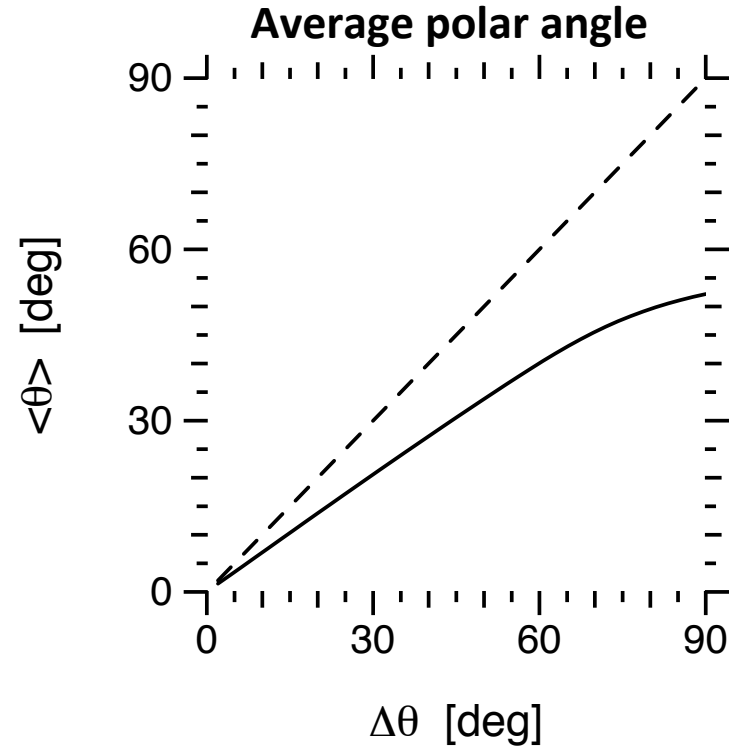
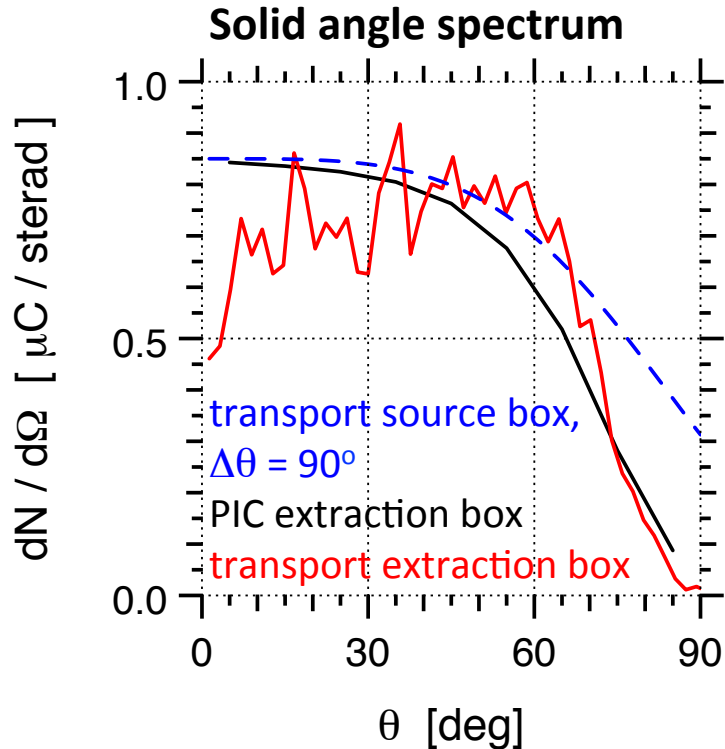
PIC fast electron angle spectrum is very divergent

solid angle spectrum in source box:

$$\frac{dN}{d\Omega} = \exp\left[-(\theta / \Delta\theta)^4\right]$$



$\Delta\theta = 90^\circ$ needed to agree with PIC results



$\Delta\theta$	$\langle\theta\rangle$	Zuma-Hydra runs used for
10°	6.9°	artificially collimated source
90°	52°	realistic PIC source

Zuma: Hybrid PIC code (D. J. Larson)

- Field and background dynamics simplified to eliminate light and plasma waves:

$$\text{valid for } \omega \ll \omega_{\text{plasma}}, \omega_{\text{laser}} \quad k \ll k_{\text{laser}}, 1/\lambda_{\text{Debye}}$$

- Relativistic fast electron particle push: $\vec{F} = -e(\vec{E} + \vec{v} \times \vec{B})$
- Fast e- energy loss (drag) and angular scattering: formulas of Solodov, Davies
- $\vec{J}_{\text{return}} = -\vec{J}_{\text{fast}} + \mu_0^{-1} \nabla \times \vec{B}$ Ampere's law without displacement current
- Electric field given by massless momentum equation for background electrons:

$$m_e \frac{d\vec{v}_{eb}}{dt} = -e\vec{E} + \dots = 0$$

$$\rightarrow \vec{E} = \vec{\eta} \cdot \vec{J}_{\text{return}} - e^{-1} \vec{\beta} \cdot \nabla T_e - \frac{\nabla p_e}{en_{eb}} - \vec{v}_{eb} \times \vec{B}$$

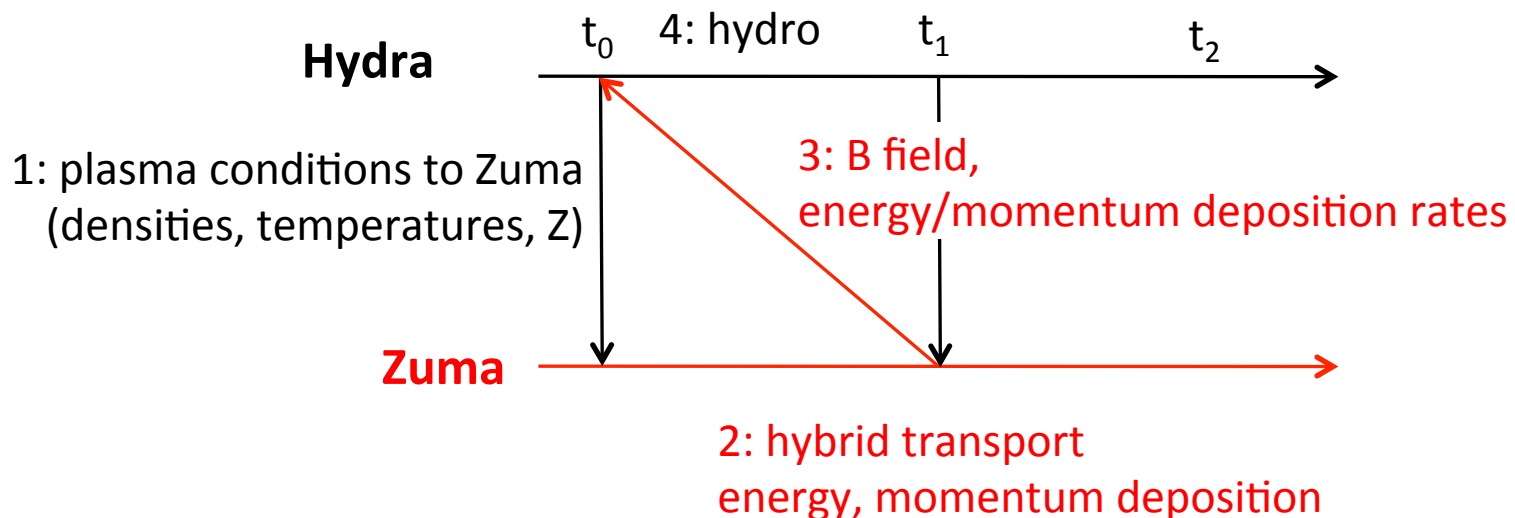
η = resistivity from Lee-More-Desjarlais

- $\vec{E} = \eta \vec{J}_{\text{return}}$ Simple Ohm's law, used for this talk's runs
- $\vec{J}_{\text{return}} \cdot \vec{E}$ Ohmic heating
- $\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E}$ Faraday's law

Hybrid PIC transport code Zuma coupled to rad-hydro code Hydra

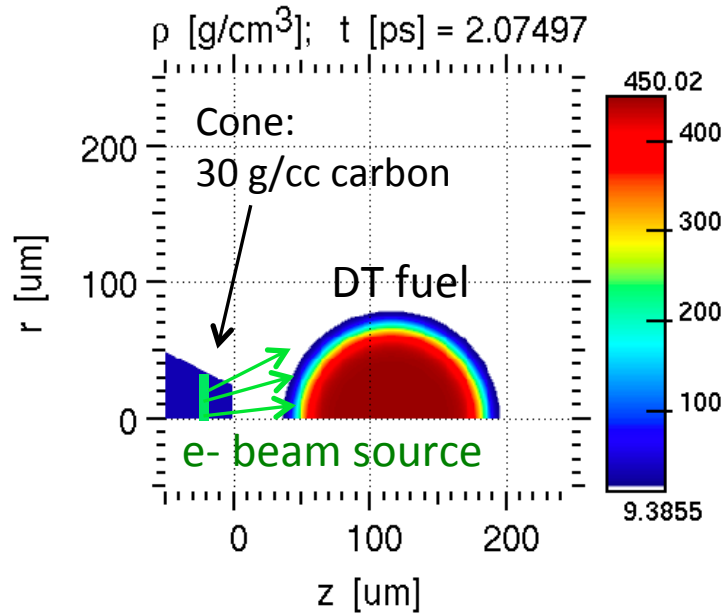
(M. M. Marinak, D. J. Larson, L. Divol)

- Both codes run in cylindrical R-Z geometry on fixed Eulerian meshes (which can differ)
- Data transfer done via files generated by Overlink code [J. Grandy et al.]
- Typical run: 20 ps transport (Zuma + Hydra), then 180 ps burn (just Hydra)
 - 2-3 wall-time hours on 40 cpu's

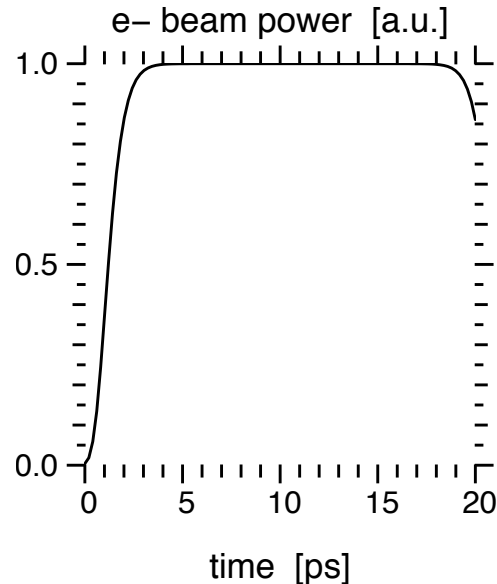
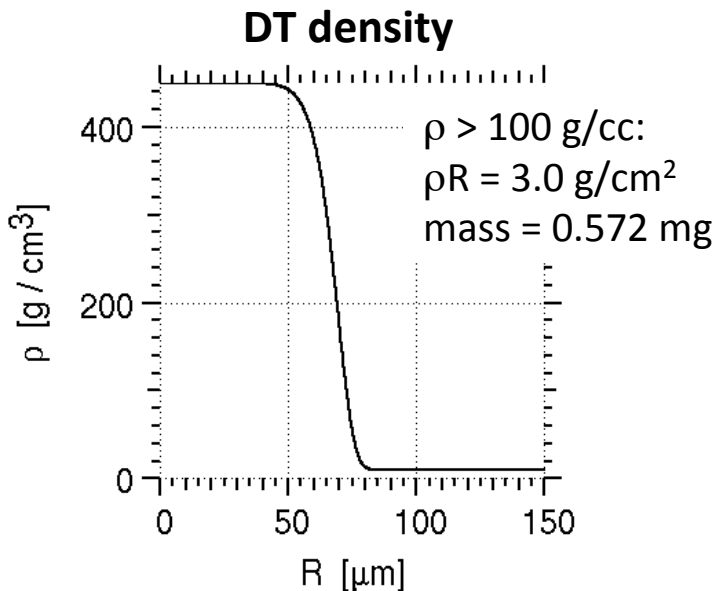


- Hydra details: IMC photonics, neutron deposition turned off, no MHD package used

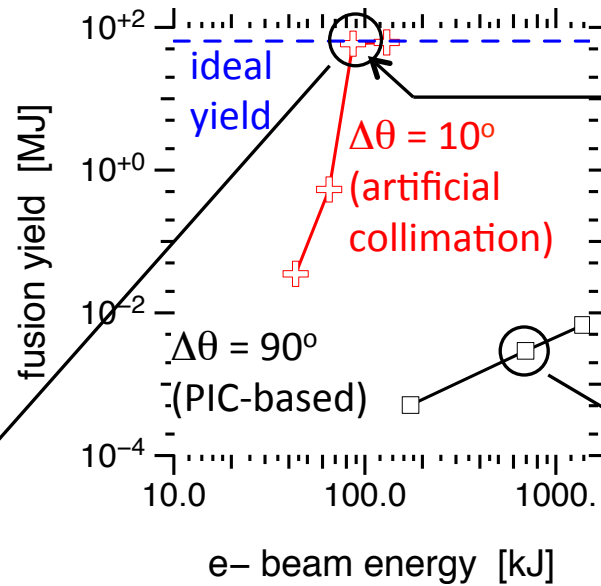
ignition-scale toy target with carbon cone



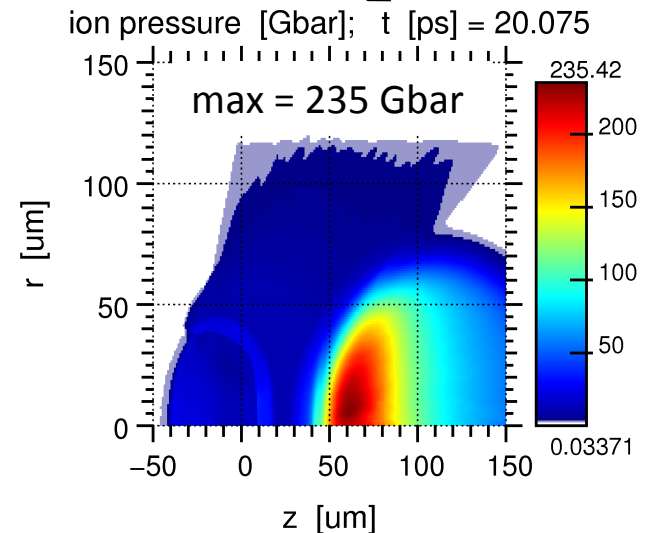
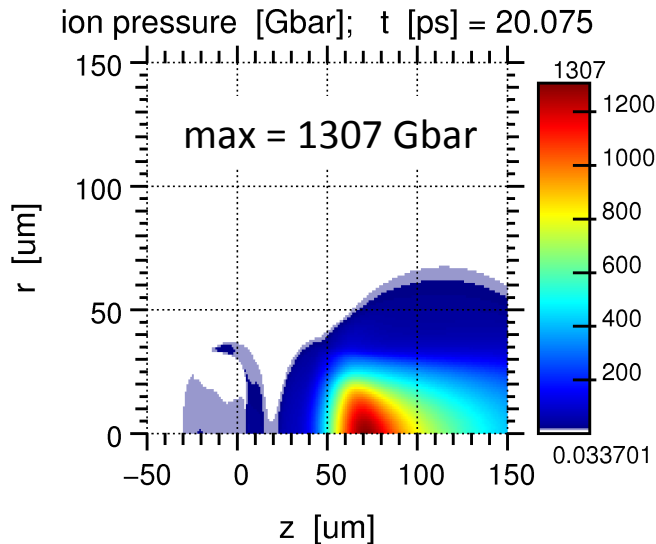
- Ideal burn-up fraction $f = \rho R / (\rho R + 6) = 1/3$
- Ideal fusion yield = 338 MJ * Mass [mg] * $f = 64.4$ MJ
- Optimal e-beam ignition energy [Atzeni et al., PoP 2007]
 $140 \text{ kJ} / (\rho / 100 \text{ g/cc})^{1.85} = 8.7 \text{ kJ}$
- Beam intensity = $I_0 \exp(-0.5 * (r/23)^8)$
HWHM: $r = 24 \mu\text{m}$
- 527 nm (2ω) wavelength laser: lowers $T_{\text{pond}} \sim \lambda$



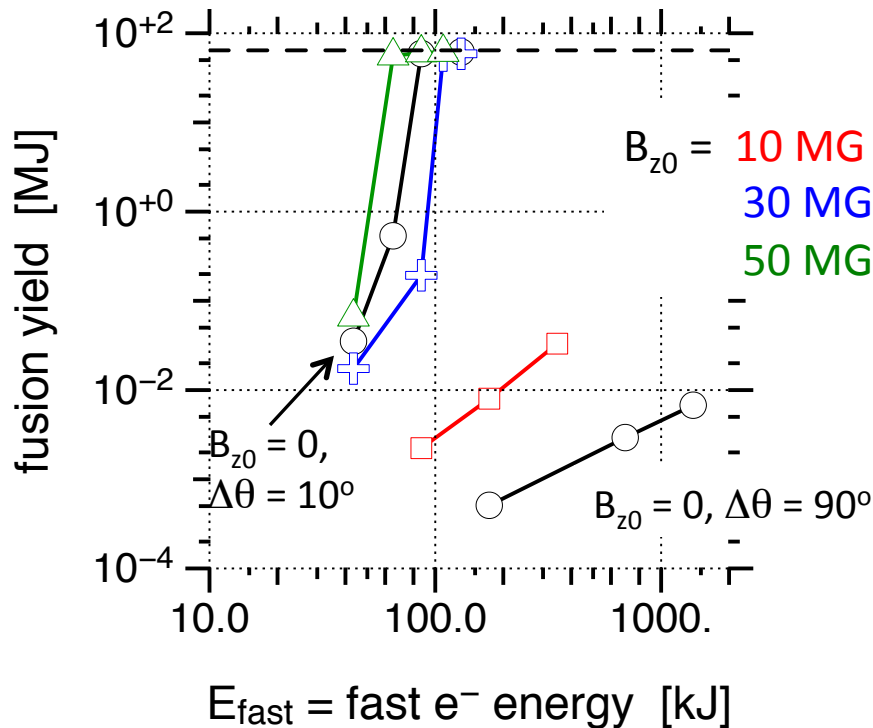
PIC-based source divergence gives prohibitive ignition energies; dramatically reduced if source is artificially collimated



87 kJ ignition energy
10x ideal value:
divergence, spectrum too energetic, pulse and spot un-optimized



Adding an initial, uniform, axial magnetic field B_z reduces ignition energy to that of artificially collimated beam



e- Larmor radius:

$$r_{Le} \propto \frac{\gamma\beta}{B} = \frac{33.4 \mu\text{m}}{B_{MG}} \left[W_{MV}^2 + 1.02 W_{MV} \right]^{1/2}$$

For a 2 MeV e- (roughly optimal to deposit energy in hot spot), $r_{Le} = 82 \mu\text{m} / B \text{ [MG]}$

$r_{Le} = \text{spot radius (24 } \mu\text{m)}$ for $B = 3.4 \text{ MG}$:
 lower bound on when B fields matter

Rad-hydro-MHD studies of B field compression have been started by H. D. Shay, M. Tabak

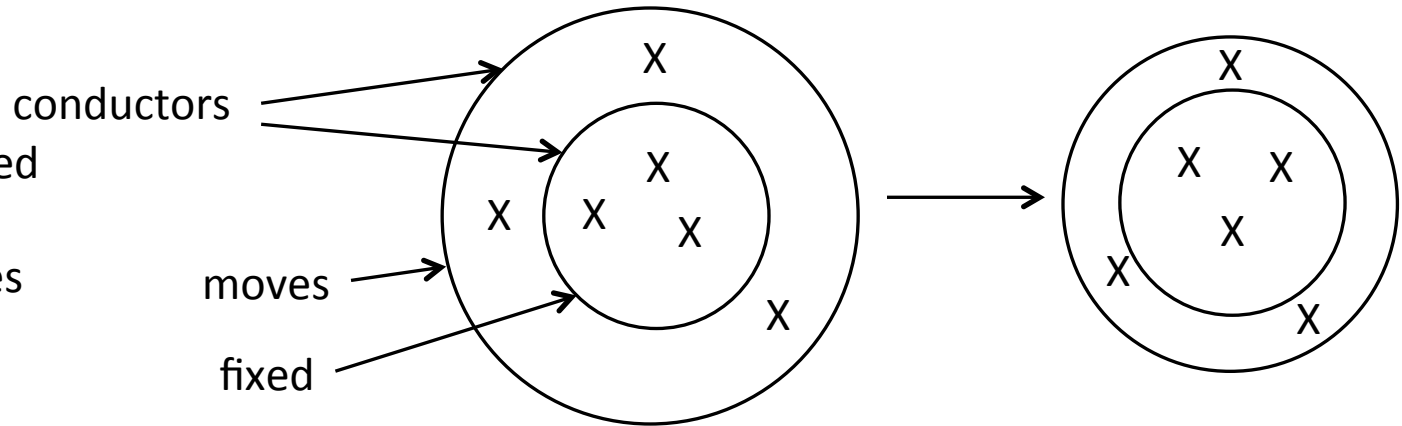
Omega experiments show compression of 50 kG seed B field in cylindrical implosions¹ to 30-40 MG, and in spherical implosions² to 20 MG

¹J. P. Knauer, Phys. Plasmas 17, 056318 (2010)

²P. Y. Chang et al., talk J05-2, APS-DPP 2010

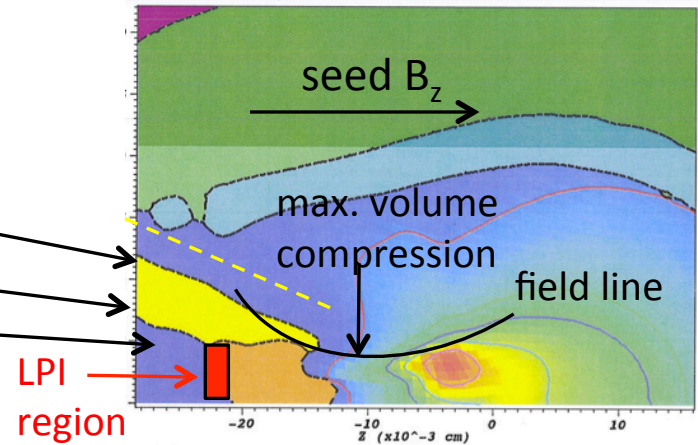
Implosion can compress magnetic field in DT, but short-pulse LPI will likely happen in the seed field

Flux $B_z * r^2$ conserved inside conductor, unless field diffuses due to resistivity

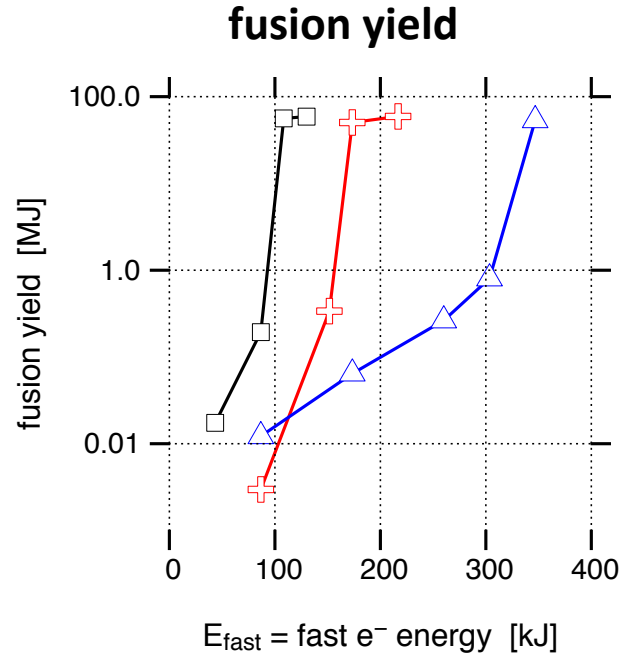
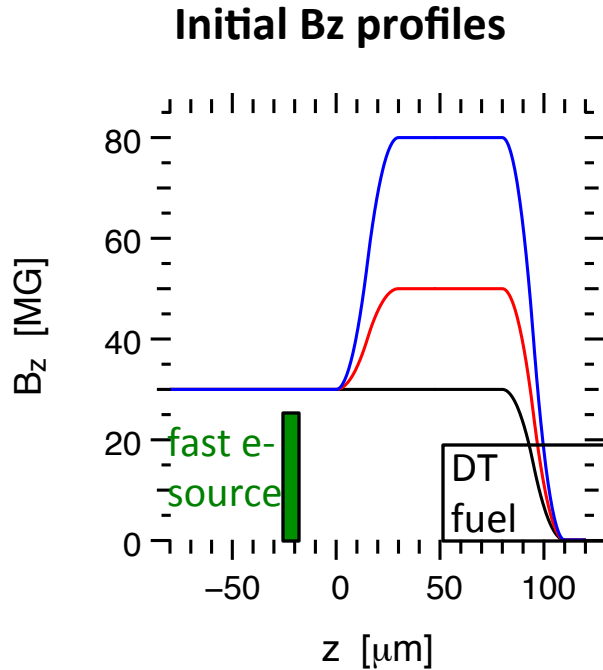


Nested conductors:
Field compressed between conductors, but not inside inner one

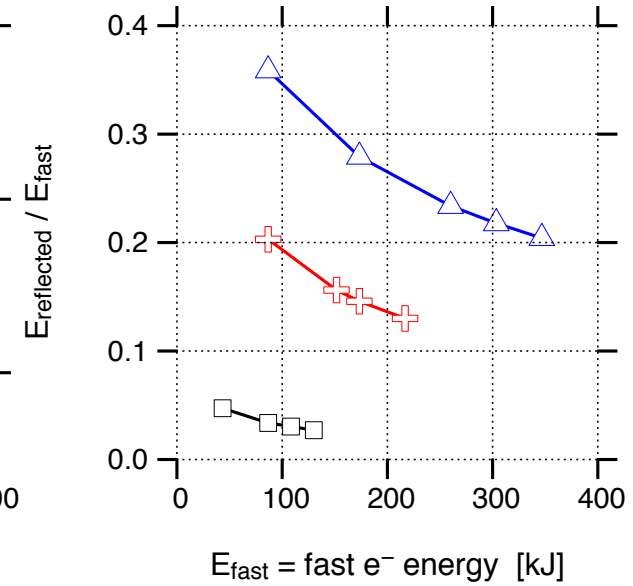
- Cone outer surface compressed
- Cone inner surface doesn't move – shock break-out would fill cone
- B field in vacuum region is uncompressed; may increase over seed field due to diffusion



Axial variation in magnetic field strength reduces hot-spot heating due to magnetic mirroring



Fast e- energy reflected to left boundary



$$B_z(r, z) = B_{z0} + (B_{z1} - B_{z0})G(r)H(z)$$

$$H(z) = \left(1 + \left(\frac{z - z_0}{\Delta z} \right)^2 \right)^{-1}$$

$$G(r) = \exp[-(r / 50 \mu\text{m})^8]$$

$$B_r(r, z) \text{ from } A_\phi \text{ to ensure } \nabla \cdot \vec{B} = 0$$

Axial variation in B_z gives rise to B_r , to satisfy $\text{div } B = 0$.
Leads to mirror force in z direction.

Magnetic mirroring in cylindrical B field

$$\frac{d\vec{p}}{dt} = q\vec{v} \times \vec{B} \quad \vec{B} = B_z(z)\hat{z} + B_r(r,z)\hat{r} \quad \omega_{cz} = \frac{qB_z}{\gamma m}$$

constants of motion: γ and canonical

angular momentum $l_0 = r^2(\dot{\phi} + \omega_{cz}/2)$

$$\text{Radial motion: } \frac{d^2r}{dt^2} = -\frac{d}{dr}U_r \quad U_r = \frac{l_0^2}{2r^2} + \frac{r^2}{8}\omega_{cz}^2$$

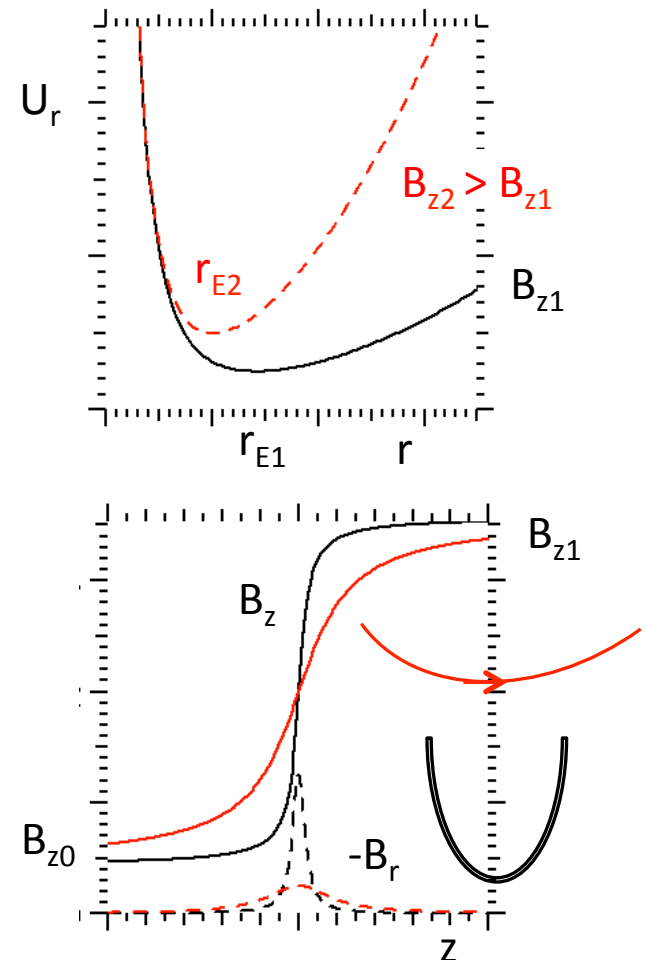
$$\text{equilibrium: } dU_r/dr = 0 \quad \rightarrow \quad r_E = \left| \frac{2l_0}{\omega_{cz}} \right|^{1/2}$$

$$\text{Axial motion: } \frac{d^2z}{dt^2} = \frac{1}{8}(\sigma r_E^2 - r^2)\frac{\partial}{\partial z}\omega_{cz}^2 \quad \sigma = \text{sign}(l_0\omega_{cz})$$

Mirroring in increasing B_z

- Fight between dB_z/dz and decreasing radius
- Adiabatic invariant for slowly changing B_z :
"loss cone" $\tan^2\theta_L = B_{z0}/(B_{z1}-B_{z0})$
- Loss cone not accurate for rapidly-varying B_z :
Large B_r can mirror particles with $v_{\text{perp}}(t=0) = 0$
(always in adiabatic loss cone)

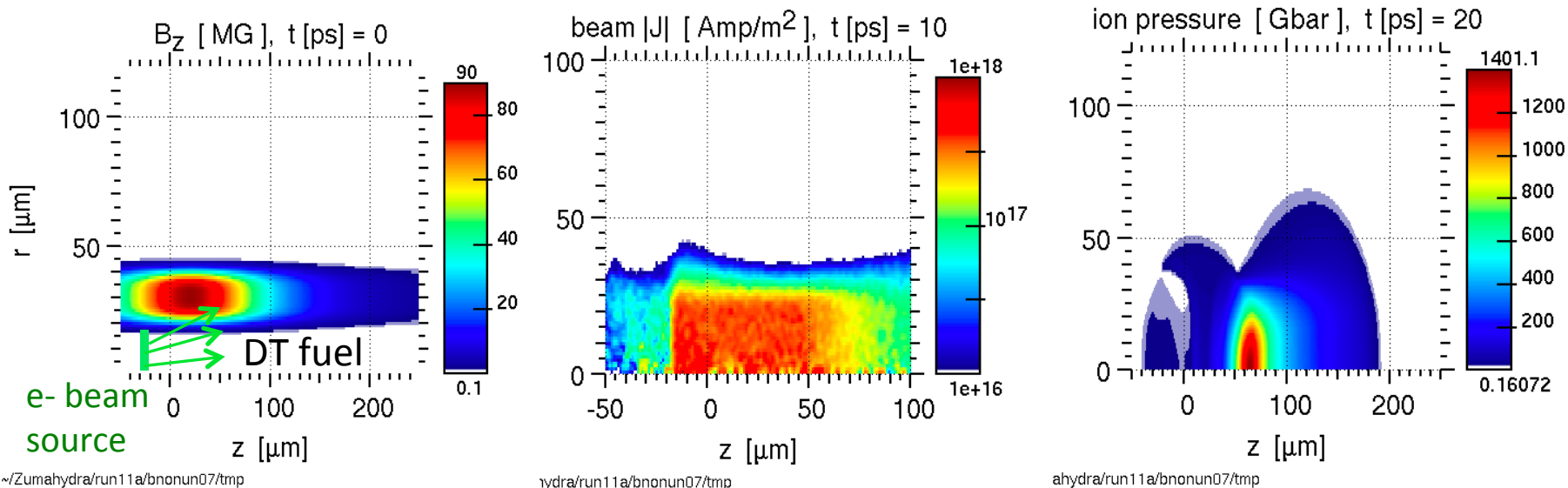
$$\nabla \cdot \vec{B} = 0 \quad \rightarrow \quad B_r = -\frac{r}{2} \frac{\partial B_z}{\partial z}$$



We can circumvent mirroring with “magnetic pipe:”

B_{z0} peaks off-axis

- Run with $B_{z0} = 90$ MG, $E_{\text{beam}} = 87$ kJ ignites
- Using $B_{z0} = 60$ MG, or narrower in z , or $E_{\text{fast}} = 43.4$ kJ all fail (<270 kJ fusion yield).
- Artificially collimated beam ($\Delta\theta = 10^\circ$) requires $E_{\text{fast}} = 87$ kJ to ignite.



Very little backward-going e-,
unlike mirroring cases

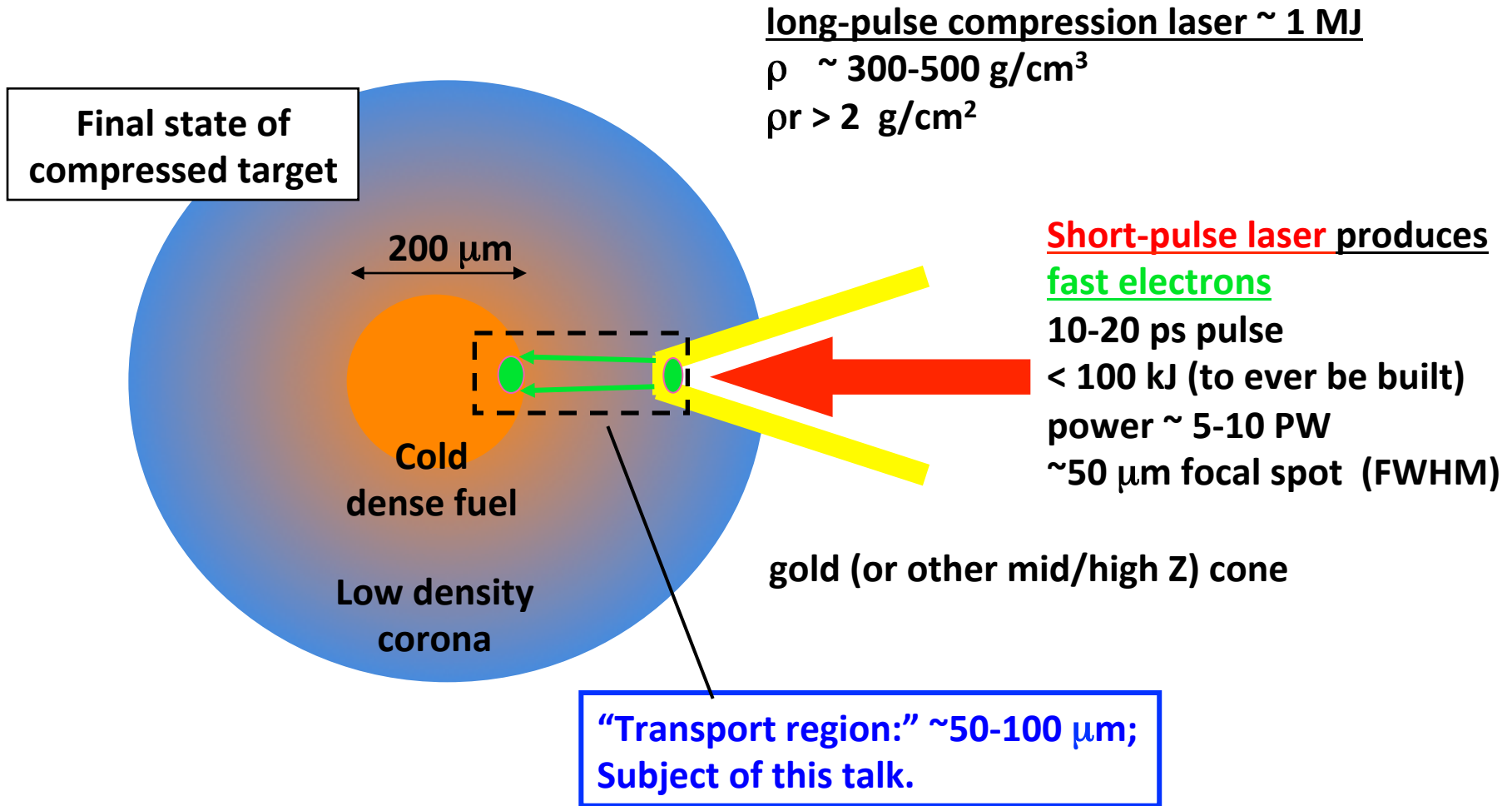
Imposed magnetic fields may circumvent large fast-electron divergence for fast ignition, but mirroring is an issue

- Artificially collimated e- beam: ignition $E_{\text{fast}} = 87 \text{ kJ}$
- Realistic PIC beam divergence: ignition $E_{\text{fast}} \sim \text{MJ's}$
- Uniform initial axial magnetic field $> 30 \text{ MG}$: ignition $E_{\text{fast}} = 100 \text{ kJ}$
- Non-uniform field peaking in fuel: fast e- reflected by mirror force
- Magnetic pipe: hollow radial profile: can recover ignition $E_{\text{fast}} = 87 \text{ kJ}$

How can we assemble such fields in an implosion?

-
- **Backup slides beyond here**

We are pursuing fast ignition for high gain and inertial fusion energy

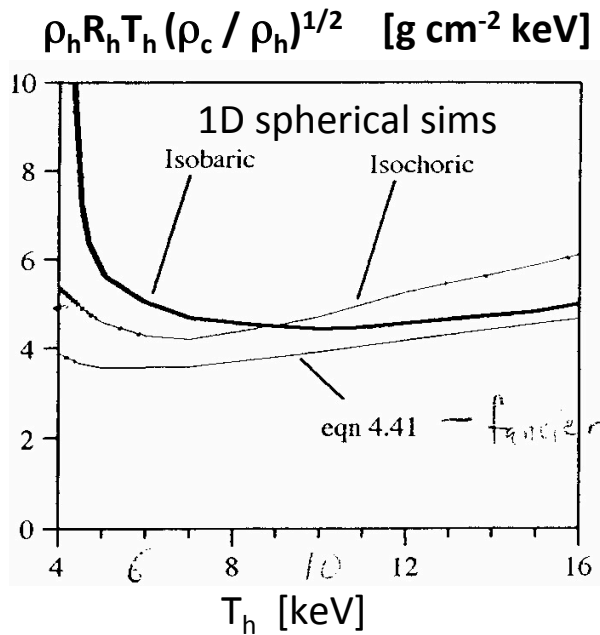


M. Tabak, J. Hammer, M. E. Glinsky, et al., *Phys. Plasmas* **1**, 1626 (1994).

Isochoric ignition hot-spot: $T_{\text{ion}} > 4 \text{ keV}$ and $\rho^*R^*T_{\text{ion}} > 5 \text{ g cm}^{-2} \text{ keV}$

Atzeni and Meyer-ter-Vehn: *The Physics of Inertial Fusion*: p. 85.

X_h = hot-spot value; ρ_c = density of surrounding cold fuel. $\rho_c = \rho_h$ for isochoric.



$(\rho_c / \rho_h)^{1/2} = (1, 3-4)$ for (isochoric, NIC isobaric).

Isobaric ignition requires a smaller hotspot $\rho_h R_h T_h$ but more laser energy to achieve a larger ρ_c .

$\rho^*R^*T_{\text{ion}} = \text{max.}$ at end of e- source pulse, centered on peak ion pressure.

We can circumvent mirroring with “magnetic pipe:”

B_{z0} peaks off-axis

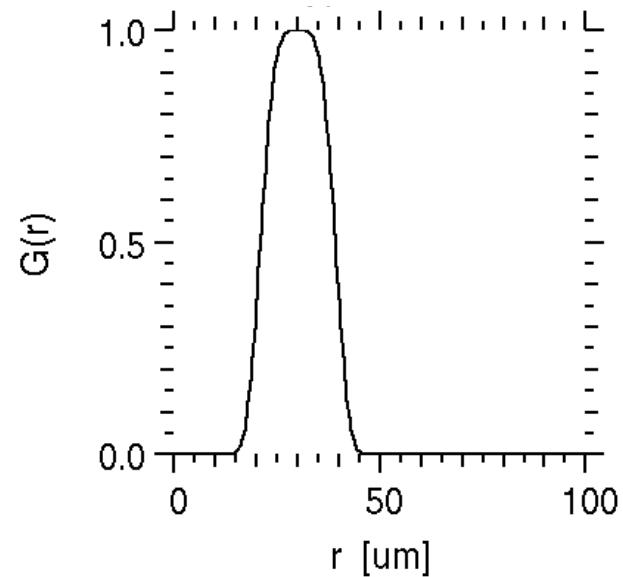
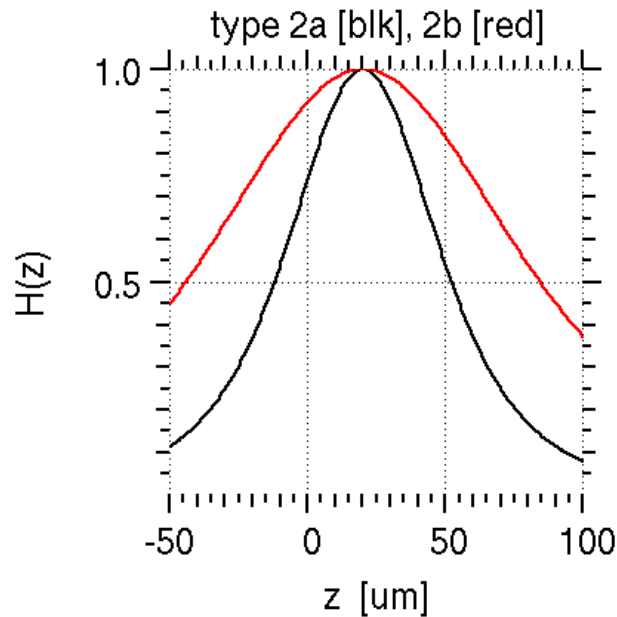
$$B_z(r, z) = B_{z0} + (B_{z1} - B_{z0})G(r)H(z)$$

$$G(r) = \exp\left[-\left(\frac{r - r_0}{\Delta r}\right)^4\right]$$

$$H(z) = \left[1 + \left(\frac{z - z_0}{\Delta z}\right)^2\right]^{-2}$$

Field type 2a: $B_{z0} = 0.1$ MG, B_{z1} varies,
 $z_0 = 20$, $\Delta z = 50$, $r_0 = 30$, $\Delta r = 10$

Field type 2b: same as 2a, but $\Delta z = 100$



Magnetic field evolution governed by MHD frozen-in law

$$\partial_t \vec{B} = -\nabla \times \vec{E}$$

$$\vec{E} = -\vec{v}_e \times \vec{B} + \eta \vec{J}_e$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}_e$$

$$\longrightarrow \frac{\partial}{\partial t} r B_z + \frac{\partial}{\partial r} v_r r B_z = \mu_0^{-1} \frac{\partial}{\partial r} \left[r \eta \frac{\partial B_z}{\partial r} \right]$$

Cylindrical geometry:

$$\vec{B} = B_z(r, t) \hat{z}$$

magnetic flux $\psi \equiv \int_{r_1}^{r_2} da \hat{z} \cdot \vec{B} = 2\pi \int_{r_1}^{r_2} dr r B_z$

$$\eta = \eta(r, t)$$

$$\vec{v}_e = v_r(r, t) \hat{r}$$

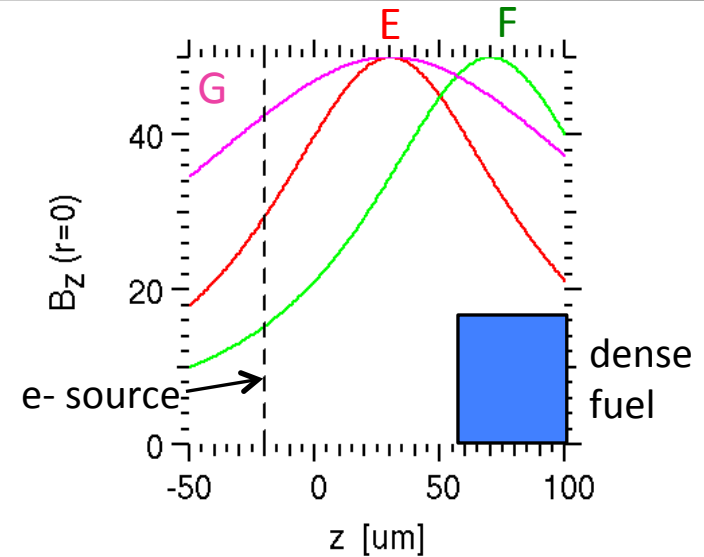
Let $\frac{dr_i}{dt} = v_r(r_i, t)$ follows plasma electron flow

$$\text{Then } \frac{d\psi}{dt} = \frac{2\pi}{\mu_0} \left(r \eta \frac{\partial B_z}{\partial r} \right) \Big|_{r_1}^{r_2}$$

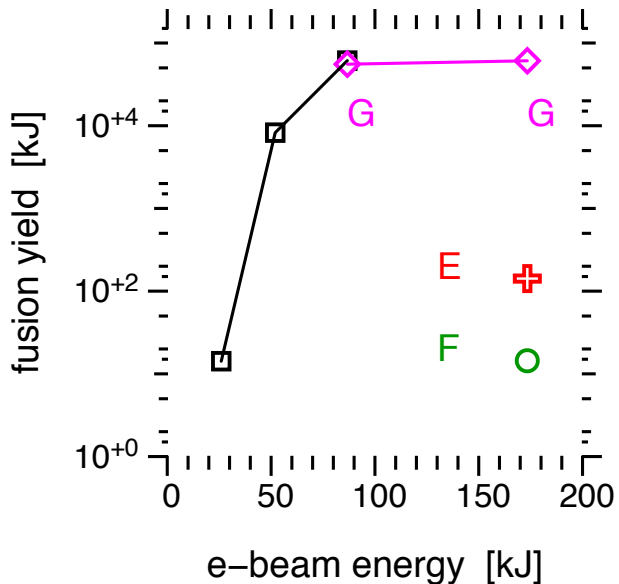
Frozen-in law: magnetic flux between two surfaces moving with the plasma electrons changes only due to magnetic diffusion.

Mirroring with non-uniform imposed B-fields: effective beam energy partly follows mirror scaling

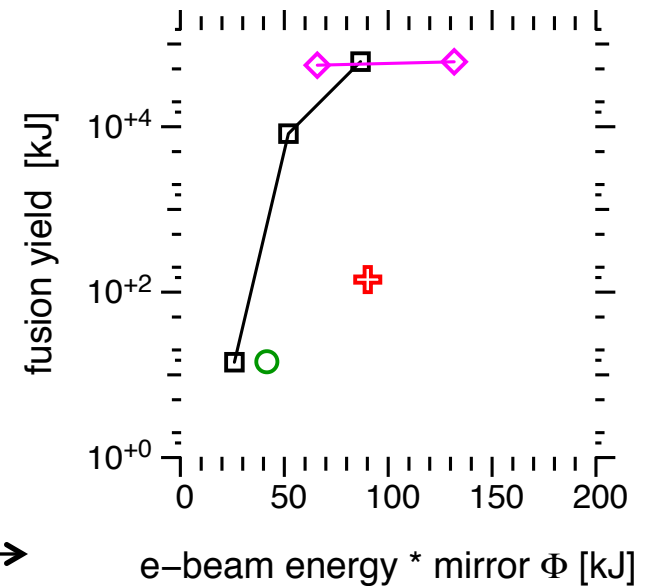
B field type	$B_{z,\text{fuel}} / B_{z,\text{exc}}$	mirror Φ
E	0.66	0.52 (mid)
F	0.34	0.24 (worst)
G	0.88	0.76 (best)



black: uniform $B_{z0}=50 \text{ MG}$; mirror $\Phi = 1$.

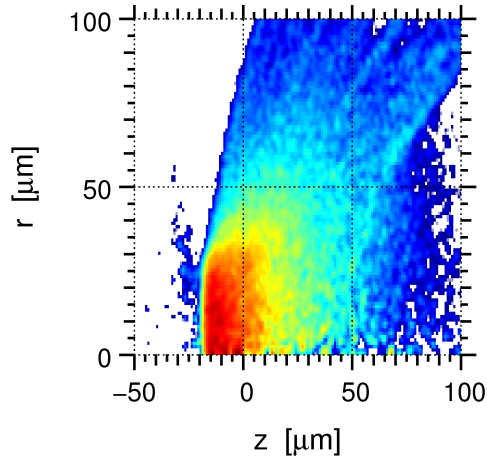


* mirror Φ →

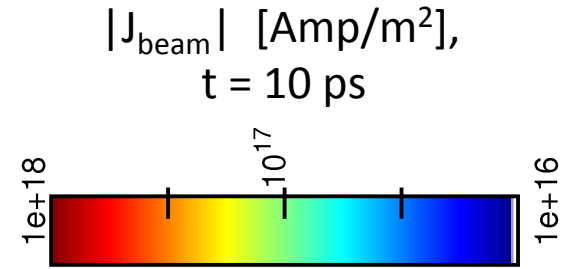
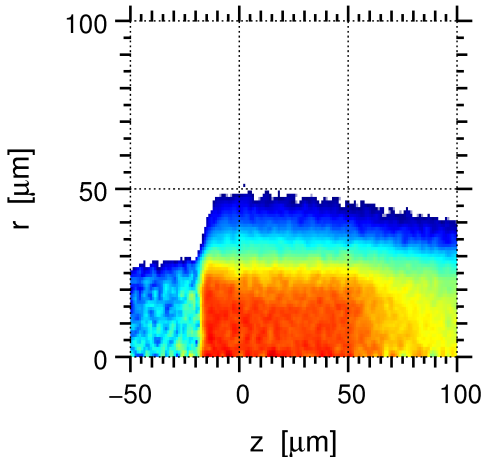


Evidence of mirroring with non-uniform imposed B-fields: reflected fast electrons

$B_{z0} = 0, E_{\text{beam}} = 173 \text{ kJ}$



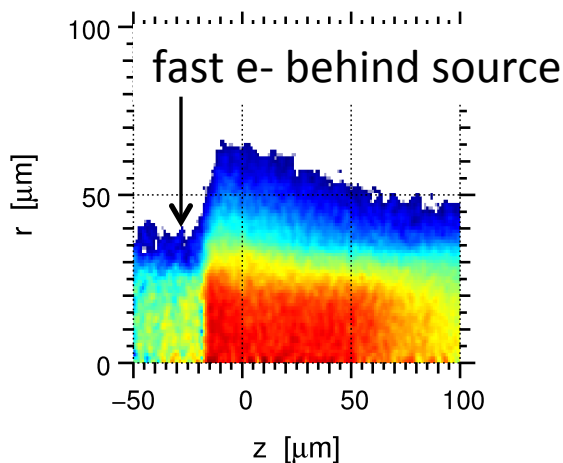
$B_{z0} = 50 \text{ MG}, E_{\text{beam}} = 87 \text{ kJ}$



more mirroring

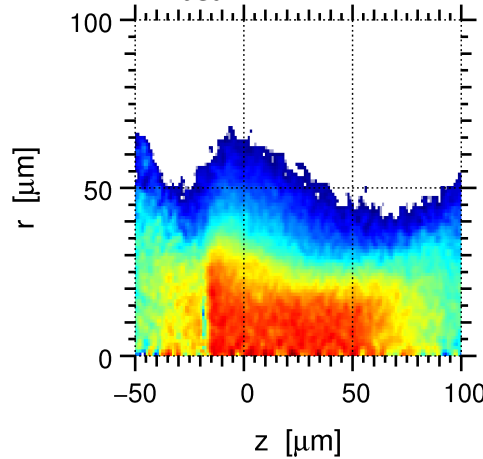
field type G,

$E_{\text{beam}} = 173 \text{ kJ}$



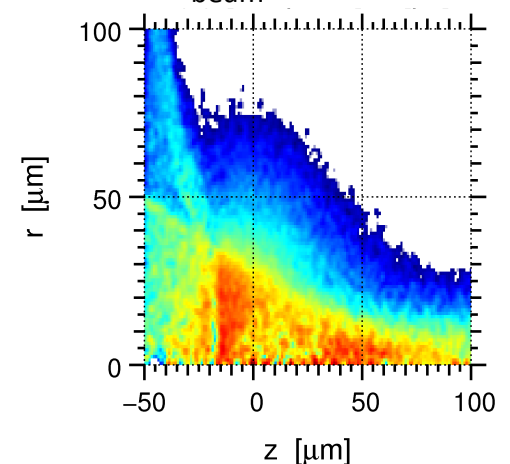
field type E,

$E_{\text{beam}} = 173 \text{ kJ}$



field type F,

$E_{\text{beam}} = 173 \text{ kJ}$



Magnetic mirroring generalities (fully relativistic)

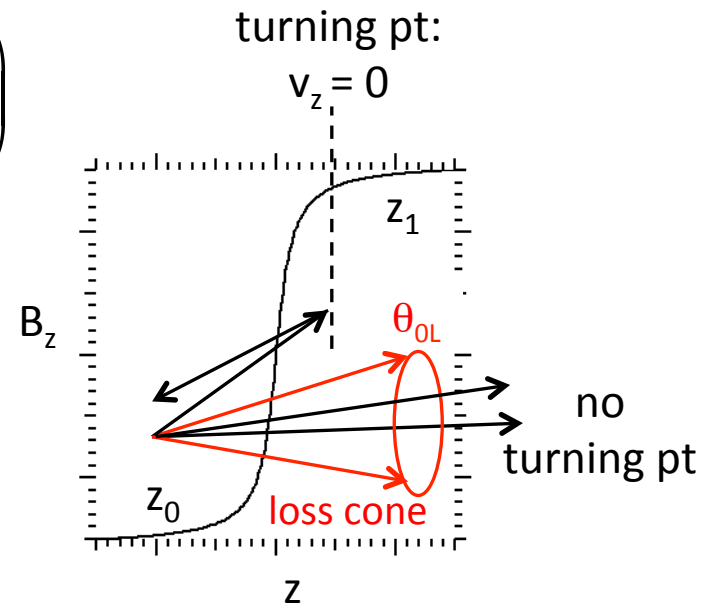
- $\text{div } \mathbf{B} = 0$ implies $B_r(r,z) = -(r/2) dB_z / dz$
- Mirroring due to z force on a particle: $F_z = q v_{\perp} \times B_r$
- Adiabatic limit: $\left| \frac{1}{B} \frac{dB}{dt} \right| \ll \text{cyclotron freq.}$
- Magnetic moment = adiabatic invariant: *not* exactly conserved, but change is small

$$\mu = \oint \vec{p}_{\perp} \cdot d\vec{l} = \pi \frac{c}{e} \frac{p_{\perp}^2}{B_z} \quad \rightarrow \quad \frac{v_{\perp}^2}{B_z} = \text{const.}$$

$$v_z^2 + v_{\perp}^2 = \text{const.} \quad \rightarrow \quad v_z^2 = v_{z0}^2 + v_{\perp 0}^2 \left(1 - \frac{B_z}{B_{z0}} \right)$$

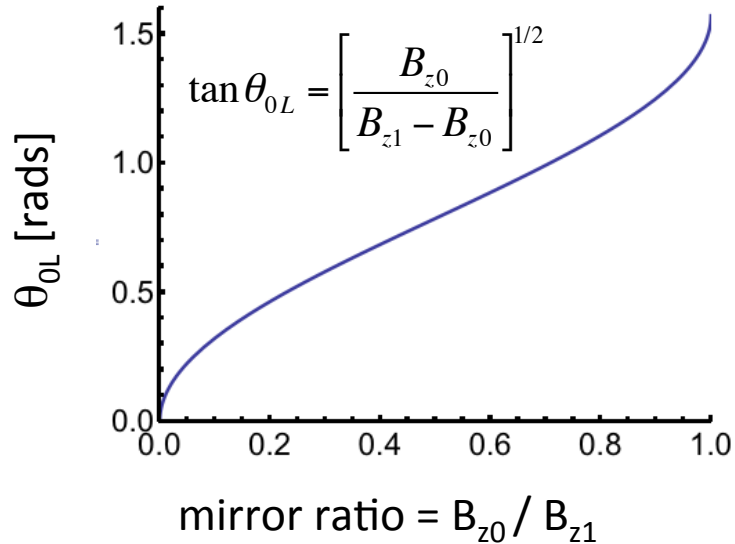
- Loss cone bad in MFE mirror machine,
but good for us: these e- reach the fuel

$$\text{loss cone: } \tan \theta_{0L} = \frac{v_{\perp}}{v_z} = \left[\frac{B_{z0}}{B_{z1} - B_{z0}} \right]^{1/2}$$



Mirroring with our electron source

Loss cone angle vs. mirror ratio



- e- source: $\frac{d^2N}{dEd\theta} = \frac{dN}{dE} \cdot \frac{dN}{d\theta}$

$$\frac{dN}{d\theta} = \sin \theta \exp \left[-(\theta / 90 \text{ deg.})^4 \right]$$

Number in loss cone: $F(\theta) = \int_0^\theta d\theta \frac{dN}{d\theta}$

loss-cone fraction: $\Phi = \frac{F(\theta)}{F(\pi/2)}$

Loss cone fraction of our $dN/d\theta$ vs. mirror ratio

