Nonlinear Landau Damping Rate of a Driven Plasma Wave

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In this Letter, we discuss the concept of the nonlinear Landau damping rate, ν, of a driven electron plasma wave, and provide a very simple, practical formula for ν, which agrees very well with results inferred from Vlasov simulations of stimulated Raman scattering. ν actually is more complicated an operator than a plain damping rate, and it may only be seen as such because it assumes almost constant values before abruptly dropping to 0. The decrease of ν to 0 is moreover shown to occur later when the wave amplitude varies in the direction transverse to its propagation.

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As is well known, in a collisionless plasma, and in the linear regime, an electron plasma wave (EPW) with phase velocity v_ϕ accelerates the electrons of initial velocity $v_0 < v_\phi$, and decelerates the other ones. If this leads to an overall acceleration of the electrons by the wave, as, for example, in an initially Maxwellian plasma, then because of energy (or momentum) conservation, the plasma wave damps at a rate, ν_L, first derived by Landau in his famous 1946 paper, Ref. [1]. Landau damping is therefore a noncollisional, linear phenomenon, which is actually primarily due to the nearly resonant electrons, those such that $|v_0 - v_\phi| \lesssim \nu_L/k$, where k is the plasma wave number.

A nonlinear counterpart of ν_L was first calculated by O'Neil in Ref. [2], who considered an electron plasma wave of constant and uniform amplitude, E_0, which grew infinitely quickly in an initially Maxwellian plasma. When $\omega_B \gg \nu_L$, where $\omega_B = \sqrt{ekE_0/m}$, $-e$ being the electron charge and m its mass, most of the nearly resonant electrons are trapped and oscillate in the wave trough. Within one oscillation period, a trapped electron neither gains nor loses momentum in the wave frame, so that the mechanism which gave rise to Landau damping vanishes, and so does the damping rate after a few oscillations at a frequency close to ω_B, as shown by O'Neil.

Despite the countless number of papers written on the subject since these two seminal works were published, we are not aware of any simple analytic expression, supported by numerical simulations, for the nonlinear noncollisional damping rate of an EPW whose amplitude may arbitrarily vary in space and time. This is what we provide in this Letter, in the limit of nonrelativistic electron motion and slow variation of the wave amplitude. We moreover focus on driven plasma waves since our work was primarily motivated by recent numerical [3,4] and experimental [5] papers on stimulated Raman scattering (SRS) reporting reflectivities far above what could be inferred from linear theory, with direct implication to inertial confinement fusion. This so-called "kinetic enhancement" was attributed to the nonlinear reduction of the Landau damping rate, although no theory, nor analytic formula, was available to support this assumption. The present Letter fills this gap.

Before proceeding, it is necessary to clarify what one means by the "nonlinear Landau *damping* rate" of a wave which, since it is driven, grows. Actually, the driven EPW accelerates electrons exactly the same way as if it were freely propagating, which hampers its growth, and one would like to account for this through an effective damping rate that could be used in an envelope equation. More precisely, when the EPW and driving electric fields are, respectively, $\vec{E}_{\rm EPW} = E_p \sin\varphi \hat{x}$ and $\vec{E}_{\rm drive} = E_d \cos(\varphi + \delta\varphi)\hat{x}$, with $|E^{-1}_{p,d}\partial_x E_{p,d}| \ll \partial_x\varphi \equiv k$, $|E^{-1}_{p,d}\partial_t E_{p,d}| \ll -\partial_t\varphi \equiv \omega$, and $\delta\varphi \ll \varphi$, one would like to write the following envelope equation for the EPW amplitude,

$$\partial_t E_p + v_g \partial_x E_p + \nu E_p = E_d \cos(\delta\varphi)/\partial_\omega \chi^r_{\rm env} \quad (1)$$

where ν is called the (nonlinear) Landau damping rate of the driven plasma wave. Actually, the nonlinear envelope equation of an EPW has already been derived in Ref. [6], and is, when $\mathrm{Re}(\chi) \approx -1$ and $|\mathrm{Im}(\chi)| \ll 1$,

$$\mathrm{Im}\,(\chi)E_p - k^{-1}\partial_x E_p = E_d \cos(\delta\varphi) \quad (2)$$

where χ is the electron susceptibility, $\chi \equiv i\rho_0/(\varepsilon_0 k E_0)$, ρ_0 and E_0 being, respectively, the complex amplitudes of the charge density, ρ, and of the total longitudinal electric field, i.e., $E_{\rm EPW} + E_{\rm drive} \equiv E_0 e^{i\varphi}$ + c.c. and $\rho \equiv \rho_0 e^{i\varphi}$ + c.c. In this Letter, we derive a theoretical expression for $\mathrm{Im}(\chi)$ showing that Eq. (2) can indeed be cast in the form of Eq. (1) and provide explicit formulas for all the coefficients of this equation. The accuracy of our theoretical estimate for $\mathrm{Im}(\chi)$ can be appreciated in Fig. 1(a), while the nonlinear variations of the coefficients of Eq. (1) are illustrated in Figs. 1(b)–1(d).

Let us first assume that the total longitudinal field amplitude, E_0, is uniform, while its time variation is such that $|\Gamma| \equiv |E_0^{-1} d_t E_0| \ll \omega_{\rm pe}$, where $\omega_{\rm pe}$ is the plasma frequency. For a freely propagating plasma wave, when $\nu_L \ll \omega_{\rm pe}$, the Landau damping rate may be estimated using

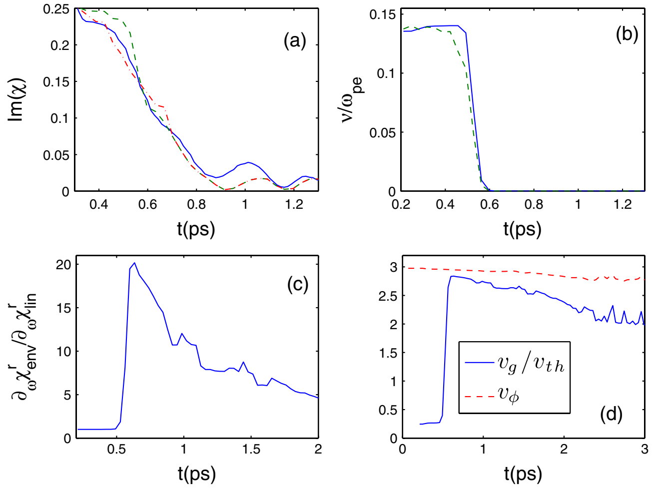

FIG. 1 (color online). Panel (a), $\mathrm{Im}(\chi)$ calculated numerically (blue solid line) and theoretically using for $\mathrm{Im}(\chi_{\mathrm{per}})$ a 1st order (green dashed line) or an 11th order (red dashed-dotted line) perturbation analysis; panel (b), the nonlinear Landau damping rate normalized to the plasma frequency from a 1st order (blue solid line) or an 11th order (green dashed line) perturbation analysis; panel (c), $\partial_\omega \chi^r_{\mathrm{env}}$ normalized to its linear value and, panel (d), the EPW group velocity (blue solid line) and phase velocity (red dashed line) normalized to the thermal one.

the expansion $\mathrm{Im}[\chi(\omega - i\nu_L)] \approx \mathrm{Im}[\chi(\omega - i0)] - \nu_L \partial_\omega \mathrm{Re}(\chi)$. Then, $\mathrm{Im}(\chi) = 0$ yields $\nu_L = \mathrm{Im}[\chi(\omega - i0)]/\partial_\omega \mathrm{Re}(\chi)$. Here, we would like to make a similar expansion to get

$$\mathrm{Im}[\chi(\omega + i\Gamma)] \approx \mathrm{Im}[\chi(\omega + i0)] + \Gamma \partial_\omega \chi^r_{\mathrm{env}}. \qquad (3)$$

When $E_p \gg E_d$, which is typically the case for SRS (see Ref. [7] for a detailed discussion), $\Gamma \equiv E_0^{-1} d_t E_0 \approx E_p^{-1} d_t E_p$. Hence, plugging Eq. (3) into Eq. (2) would yield the envelope Eq. (1) with $\nu = \mathrm{Im}[\chi(\omega + i0)]/\partial_\omega \chi^r_{\mathrm{env}}$. In order to calculate $\mathrm{Im}(\chi)$, we use the expression, $\chi = -i(k\lambda_D)^{-2}\langle e^{-i\varphi}\rangle/\Phi$, derived in Ref. [6], where λ_D is the Debye length, $\Phi = eE_0/kT_e$, T_e being the electron temperature, and

$$\langle e^{-i\varphi}\rangle = \frac{1}{2\pi}\int_{-\pi}^{\pi}\int_{-\infty}^{+\infty} f(\varphi, v, t) e^{-i\varphi} d\varphi dv \qquad (4)$$

where f is the electron distribution function, and φ may be seen as a dynamical variable such that, for each electron, $d\varphi/dt = kv - \omega$, where v is the electron velocity. Let us first give an estimate of $\langle e^{-i\varphi}\rangle$ obtained through the means of a first order perturbation analysis. This amounts to using the following expansion $\varphi(x, t) = \varphi_0 + (v_0 - v_\phi)\tau + \delta\varphi$, where $\tau = k\lambda_D \omega_{\mathrm{pe}} t$, velocities are normalized to the thermal one, $v_{\mathrm{th}} \equiv \lambda_D \omega_{\mathrm{pe}}$, and, at 0 order in the time variations of v_ϕ,

$$\delta\varphi = -\int_0^\tau \int_0^u \Phi(\xi) e^{i(\varphi_0 + w\xi)} d\xi du + \mathrm{c.c.} \qquad (5)$$

where we have denoted $w \equiv v_0 - v_\phi$. As shown in Ref. [6], deeply trapped electrons do not contribute to $\mathrm{Im}(\chi)$, and one may therefore calculate $\mathrm{Im}(\chi)$ by only accounting for electrons with initial velocity $|v_0 - v_\phi| > V_l$, where $V_l \equiv \max\{0, [4\omega_B/(\pi k v_{\mathrm{th}})][1 - 3/\int_0^t \omega_B(u) du]\}$. Then, using the expansion, $\langle e^{-i\varphi}\rangle \approx \langle -i\delta\varphi e^{-i(\varphi_0 + w\tau)}\rangle$, we find

$$\langle e^{-i\varphi}\rangle = i\int_{|w|\geq V_l}\int_0^\tau\int_0^u \Phi(\xi) e^{iw(\xi - \tau)} f_0(w + v_\phi) du d\xi dw \qquad (6)$$

where f_0 is the electron distribution function in the limit $\Phi \to 0$. When Φ monotonically increases as a function of time, f_0 is the electron distribution function at $t = 0$, i.e., the unperturbed one (usually a Maxwellian). However, when Φ decreases to 0 after reaching high enough a value to induce nonlinear electron motion, perturbation analysis only makes sense if one uses for f_0 the electron distribution function in the limit $t \to +\infty$, and integrates the electron motion from $t = +\infty$ by taking advantage of the time-reversal invariance of the dynamics. Then, as explained in Ref. [6], f_0 is nearly symmetric with respect to v_ϕ in the interval $|v_0 - v_\phi| \leq \max(V_l)$ (as illustrated, for example, in Fig. 4 of Ref. [8]). This implies that once trapped, electrons never contribute to $\mathrm{Im}(\chi)$ again, even after being detrapped. Equation (6) may therefore be simplified by using for f_0 the unperturbed distribution function and by replacing V_l by $\max(V_l)$. Such a simplification will be implicitly used throughout the remainder of this Letter. In order to derive an expression similar to Eq. (3) for $\mathrm{Im}(\chi)$, we now use the decomposition $\langle e^{-i\varphi}\rangle \equiv I_1 + I_2$ with

$$I_1 = f_0'(v_\phi)\int_0^\tau\int_0^u \Phi(\xi)\int_{|w|\geq V_l} iwe^{iw(\xi - \tau)} dw d\xi du \qquad (7)$$

$$I_2 = i\int_{|w|\geq V_l}\int_0^\tau\int_0^u \Phi(\xi) e^{iw(\xi - \tau)} \times [f_0(w + v_\phi) - wf_0'(v_\phi)] dw d\xi du. \qquad (8)$$

Provided that $(d\Phi/d\tau)_{\tau=0}$ may be neglected, integrating Eq. (8) by parts yields, at first order in the time variations of Φ,

$$\mathrm{Re}(I_2) \approx 2\frac{d\Phi}{d\tau}\int_{|w|\geq V_l}\frac{f_0(w + v_\phi) - wf_0'(v_\phi)}{w^3} dw \qquad (9)$$

$$\equiv -(k\lambda_D)^2 (d\Phi/dt)(\partial\chi_1^r/\partial\omega) \qquad (10)$$

where the integral in Eq. (9) has to be taken in the sense of Cauchy's principal part when $V_l = 0$.

Setting $V_l = 0$ in Eqs. (7) and (9) just yields the linear value of $\mathrm{Im}(\chi)$. Then, χ_1^r is just the adiabatic approximation of the linear value of $\mathrm{Re}(\chi)$. As for I_1, since $\int_{-\infty}^{+\infty} iwe^{iw(\xi - \tau)} dw = 2\pi\partial_\xi \delta(\xi - \tau)$, where δ is the

Dirac distribution, one easily finds $I_1 = \pi f_0'(v_\phi)\Phi(\tau)$. Hence, in the linear limit, $\mathrm{Im}(\chi) = -\pi(k\lambda_D)^{-2}f_0'(v_\phi) + \Gamma\partial_\omega\chi_1^r$, which has the same form as Eq. (3). Therefore, Eq. (2) may indeed be cast in the same form as Eq. (1), with $\chi_{\mathrm{env}}^r = \chi_1^r$ and $\nu = -\pi(k\lambda_D)^{-2}f_0'(v_\phi)/\partial_\omega\chi_1^r$. The preceding linear value of ν is just the Landau damping rate, ν_L, in the limit $\nu_L \ll \omega_{\mathrm{pe}}$. Hence, our linear calculation is one derivation of the Landau damping rate which does not resort on complex contour deformation.

In the nonlinear regime, and when V_l^{-1} is much smaller than the typical time scale of variation of Φ, τ_ϕ, integrating Eq. (7) by parts yields

$$\mathrm{Re}\,(I_1) = f_0'(v_\phi)[4V_l^{-1}d\Phi/d\tau + O(V_l^{-3}d^3\Phi/d\tau^3)]. \tag{11}$$

Hence, when $V_l \gg \tau_\phi^{-1}$ which, for a slowly varying wave is typically the case when $\int\omega_B dt \gg 1$, $\mathrm{Re}(I_1)$ is nearly proportional to $d\Phi/d\tau$. As a consequence, $\mathrm{Im}(\chi)$ is nearly proportional to Γ and, in Eq. (1), $\nu \approx 0$. Physically, the decrease of ν towards 0 is due to the trapping of the nearly resonant electrons, which no longer contribute to ν while oscillating in the wave trough, just like in the situation considered by O'Neil. When $\int\omega_B dt \gg 1$, $\mathrm{Im}(\chi)$ may therefore be approximated by $\mathrm{Im}(\chi) \approx \Gamma\partial_\omega\chi_{\mathrm{eff}}^r$ where χ_{eff}^r is the real part of some effective susceptibility obtained by removing the contribution of the deeply trapped electrons. How to calculate $\partial_\omega\chi_{\mathrm{eff}}^r$ very accurately, without resorting to perturbation theory is explained in Ref. [6]. Note that the I_1 term which, in the linear limit, yields the damping rate ν, renormalizes the term $\partial_\omega\chi_{\mathrm{env}}^r$ in Eq. (1) when $\int\omega_B dt \gg 1$. In the strong damping limit, when $\nu_L \gg \Gamma$, $\partial_\omega\chi_{\mathrm{env}}^r$ may then increase by more than 1 order of magnitude, as illustrated in Fig. 1(c). As for the perturbative estimate $\mathrm{Im}(\chi_{\mathrm{per}})$ of $\mathrm{Im}(\chi)$, yielding Eqs. (7) and (9), it is valid provided that $\int\omega_B dt \lesssim 1$. Hence, in order to get an expression of $\mathrm{Im}(\chi)$ whatever the wave amplitude, we just need to connect the values of $\mathrm{Im}(\chi)$ obtained when $I_{\omega_B} \equiv \int\omega_B dt \lesssim 1$, and when $I_{\omega_B} \gg 1$, the following way,

$$\mathrm{Im}\,(\chi) \approx \mathrm{Im}(\chi_{\mathrm{per}})[1 - Y(I_{\omega_B})] + \Gamma\partial_\omega\chi_{\mathrm{eff}}^r Y(I_{\omega_B}) \tag{12}$$

where Y is a function rising from 0 to 1 as I_{ω_B} increases. From the preceding equation, we then derive

$$\chi_{\mathrm{env}}^r = (1 - Y)\times\chi_1^r + Y\times\chi_{\mathrm{eff}}^r \tag{13}$$

$$\nu = -Y\times(k\lambda_D)^{-2}\mathrm{Re}(I_1)/[\Phi\partial_\omega\chi_{\mathrm{env}}^r]. \tag{14}$$

To complete our calculation, we now need to provide a practical formula for I_1, simpler than Eq. (7), and to specify a choice for the function Y in Eq. (12). Let us first consider the case when Γ is a strictly positive constant, which is a relevant limit since our theory is only for slowly varying wave amplitudes. As shown in Ref. [6], when the EPW grows exponentially with time, $\Gamma\partial_\omega\chi_{\mathrm{env}}^r$ is very close to $\mathrm{Im}(\chi)$ whenever $I_{\omega_B} > 6$ and quickly diverges away from it when $I_{\omega_B} < 6$. Hence, Y must be such that $\mathrm{Im}(\chi)$ defined by Eq. (12) quickly changes from $\mathrm{Im}(\chi_{\mathrm{per}})$ to $\Gamma\partial_\omega\chi_{\mathrm{env}}^r$ when I_{ω_B} increases from a little less than 6 to a little more than 6. This is the case if we choose $Y(x) = \tanh^5[(e^{x/6} - 1)^3]$. Moreover, such a choice for Y yields an excellent agreement between the theoretical values of $\mathrm{Im}(\chi)$ and those inferred from test particles simulations (not shown here). This is therefore the choice we make in the general case. With such a choice, whenever ν is not negligible in Eq. (1), $\chi_{\mathrm{env}}^r \approx \chi_1^r$, so that Eq. (14) my be replaced by

$$\nu \approx -Y\times(k\lambda_D)^{-2}\mathrm{Re}(I_1)/[\Phi\partial_\omega\chi_1^r]. \tag{15}$$

As for I_1, when Γ is a strictly positive constant, one easily finds

$$\frac{\mathrm{Re}(I_1)}{f_0'(v_\phi)} = \Phi(\tau)\left[\pi - 2\tan^{-1}\left(\frac{V_l}{\gamma}\right) + \frac{2\gamma V_l}{\gamma^2 + V_l^2}\right] \tag{16}$$

where $\gamma \equiv \Gamma/k\lambda_D\omega_{\mathrm{pe}}$. In order to generalize the preceding formula, we use the expansion $\mathrm{Re}(I_1) = f_0'(v_\phi)[\pi\Phi(\tau) + \delta I_1]$ and find, from Eq. (16), $\delta I_1 \approx -(4/3)(V_l/\gamma)^3$ when $V_l \ll \gamma$, while when $V_l \ll \tau_\phi^{-1}$ Eq. (7) yields $\delta I_1 \approx -4(V_l^3/3)\int_0^\tau\int_0^u\int_0^\xi\Phi(\xi')d\xi' d\xi du$. Hence, while for an exponential growing wave $\gamma \equiv \Phi^{-1}d\Phi/d\tau = \Phi/\int\Phi d\tau$, we find that, when $V_l \ll \tau_\phi^{-1}$, Eq. (16) still holds in the general case provided that γ is expressed in terms of the time integral of Φ. When $V_l \gg \gamma$, Eq. (16) yields $\mathrm{Re}(I_1) \approx 4\gamma\Phi f_0'(v_\phi)/V_l$, which is the same as Eq. (11) provided that $\gamma = \Phi^{-1}d\Phi/d\tau$. Having clarified the actual meaning of γ in Eq. (16), we may generalize this equation by plugging into it

$$\gamma \equiv \frac{\Phi(\tau) - \Phi(\tau - \pi/V_l)}{\int_{\tau-\pi/V_l}^{\tau}\Phi(u)du} \tag{17}$$

which has the required properties $\gamma \approx \Phi/\int\Phi d\tau$ when $V_l \ll \tau_\phi^{-1}$, and $\gamma \approx d\Phi/d\tau$ when $V_l \gg \tau_\phi^{-1}$. Equations (15)–(17) therefore yield a practical expression for ν which, as shall be seen, is quite precise. The accuracy can even be improved by using the high order perturbative results of Ref. [6] to derive $\mathrm{Im}(\chi_{\mathrm{per}})$ and therefore $\mathrm{Re}(I_1)$. We will not show here the corresponding huge formulas, but Fig. 1(b) illustrates the improvement.

The previous results are easily generalized to account for one dimensional (1D) space variations of the EPW amplitude. Indeed, by using a Fourier expansion of the charge density then, as shown in Ref. [6], one finds

$$\mathrm{Im}\,(\chi) = \nu + \Gamma\partial_\omega\chi_{\mathrm{env}}^r - \kappa[\partial_k\chi_{\mathrm{env}}^r + \mathrm{Re}(\chi)/k] \tag{18}$$

where $\kappa \equiv E_0^{-1}\partial_x E_0 \approx E_p^1\partial_x E_p$, and ν and χ_{env}^r are still defined by Eqs. (13)–(17) except that I_{ω_B}, γ, and $\max(V_l)$ need now be evaluated in the wave frame. As a consequence, I_{ω_B} assumes its lowest values at the rear of the plasma wave packet, making Landau damping more efficient there, in agreement with the numerical results of Ref. [9]. Plugging Eq. (18) into Eq. (1), we find, provided that $[1 + \mathrm{Re}(\chi)] \approx 0$, the following expression for the EPW group velocity, $v_g = -\partial_k \chi^r_{\mathrm{env}}/\partial_\omega \chi^r_{\mathrm{env}} = \omega/k - 2/[k\partial_\omega \chi^r_{\mathrm{env}}]$. It is noteworthy that, since in the nonlinear regime $\chi^r_{\mathrm{env}} \neq \mathrm{Re}(\chi)$, $v_g \neq d\omega/dk$. Actually, since $\partial_\omega \chi^r_{\mathrm{env}}$ may reach values much larger than in the linear limit, the nonlinear value of v_g may get quite close to the EPW phase velocity, as shown in Fig. 1(d).

We now compare our theoretical calculations against direct 1D Vlasov simulations of SRS using the Eulerian code ELVIS [4]. In our numerical simulations, which are detailed in Refs. [4,7], the EPW results from the interaction of a pump laser, entering from vacuum on the left ($x = 0$) and a small-amplitude counterpropagating "seed" light wave injected on the right. Using a Hilbert transform of the fields, one can numerically calculate the ratio $[E_d \cos(\delta\varphi) + k^{-1}\partial_x E_p]/E_p$, which yields a first, numerical, estimate of $\mathrm{Im}(\chi)$. From Vlasov simulations, one can also extract the values of all the quantities, such as $I_{\omega_B}, \gamma, \ldots$, which enter our theoretical formula for $\mathrm{Im}(\chi)$. Using these values, we calculate a second, theoretical estimate, for $\mathrm{Im}(\chi)$. Both these estimates are compared in Fig. 1(a). The simulation results of Fig. 1 correspond to a plasma with electron temperature, $T_e = 5$ keV, and electron density $n = 0.1n_c$, where n_c is the critical density. The total length of the simulation box is $L = 270\lambda_l$, where $\lambda_l = 0.351$ μm is the laser wavelength, and the data of Fig. 1 were measured at $x = 154\lambda_l$. The laser intensity is $I_l = 4 \times 10^{15}$ W/cm^2 while the seed intensity is $I_s = 10^{-5} I_l$ and the seed wavelength is $\lambda_s = 0.609$ μm. As can be seen in Fig. 1(a), there is very good agreement between the theoretical and numerical values of $\mathrm{Im}(\chi)$, especially as regards the decrease of $\mathrm{Im}(\chi)$ from its linear value. Clearly, as defined by Eq. (14), ν is much more complicated an operator than a plain damping rate. However, as shown in Fig. 1(b), ν is nearly constant before abruptly dropping to 0 so that it may indeed be seen as a damping rate.

The variations of ν are very different from the oscillating result found by O'Neil because, in this Letter, we consider slowly varying waves inducing a nearly adiabatic electron motion. As a consequence, electrons with the same initial velocity are all trapped nearly simultaneously, which entails a much more efficient and sudden reduction of Landau damping than in the situation considered by O'Neil. Moreover, in a very recent paper, Ref. [10], a simple model was proposed to calculate the nonlinear damping rate of a driven plasma wave, provided that the EPW amplitude is uniform and strictly grows with time, a situation we already investigated in Ref. [6]. The theoretical results are compared against Vlasov simulations where the driving electric field is imposed, and constant. In Ref. [10], ν is calculated from Vlasov simulations using the formula, $\nu = E_d \cos(\delta\varphi)/(E_p \partial_\omega \chi^r_{\mathrm{env}}) - \Gamma$, where Γ is the EPW growth rate and where $\partial_\omega \chi^r_{\mathrm{env}}$ is derived using quasilinear theory, which yields values of $\partial_\omega \chi^r_{\mathrm{env}}$ significantly different from our nonlinear ones whenever we predict $\nu \approx 0$. Numerically (but not theoretically), ν is found in Ref. [10] to initially assume nearly constant values, which agrees with our findings, and then to oscillate in time just as Γ does, thus indicating that E_d/E_p has become nearly proportional to Γ and, therefore, that Landau damping has become negligible. From the data published in Ref. [10], we infer that this happens when $\omega_B t \gtrsim 10$, which is consistent with our predictions. Hence, our theory seems to agree with the numerical results of Ref. [10]. Moreover, when testing our former theoretical results, Ref. [6], for a growing wave, the authors of Ref. [10] actually solved Eqs. (48) and (49) of Ref. [6], which only hold for small-amplitude waves, and therefore did not take advantage of our nonperturbative formulas.

In a 3D geometry, $\mathrm{Im}(\chi)$ is just the statistical average, over all the transverse velocities $\vec{v}_\perp$, of the expression Eq. (18), where all the quantities which enter the theoretical formulas for ν and χ^r_{env} are evaluated in the frame moving at velocity $(\omega/k)\vec{x} + \vec{v}_\perp$ with respect to the laboratory frame. If the transverse extent of the EPW is much less than its longitudinal one, then it is clear that, for the same maximum wave amplitude, I_{ω_B} assumes smaller values than for a plane wave. As a consequence, linear theory is valid up to larger wave amplitudes in 1D than in 3D.

In conclusion, we derived a very precise theoretical estimate of $\mathrm{Im}(\chi)$ for a slowly varying electron plasma wave that we compared against results obtained from Vlasov simulations of SRS. From the expression of $\mathrm{Im}(\chi)$, we deduced the group velocity and the nonlinear Landau damping rate, ν, of the EPW and provided a practical expression for ν that may easily be generalized to allow for 3D space variations of the waves.

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